

### 1.1 Introduction

A reinforced concrete slab is a structural member, usually horizontal whose depth  $h$ , is small compared to their length,  $L$  and width,  $S$ . It may be supported by reinforced concrete beams, by masonry or reinforced concrete wall, by structural steel members, directly by columns.

The structural systems designed in third year (junior year) involved one-way slabs that carried load to beams, which, in turn, transmitted to column, and two way solid slabs by **method -3**, this method is no longer available in ACI code.

### 1.2 Loads

Loads that act on structures can be divided into three broad categories: **dead load**, **live load**, and **environmental loads**.

#### 1.2.1 Dead loads

- Are those that are constant in magnitude and fixed in location throughout the lifetime of the structure.
- Usually the major part of the dead load is the weight of the structure itself. This can be calculated with good accuracy from the design configuration, dimensions of the structure, and density of the material.
- For buildings, floor fill, finish floor, and plastered ceilings are usually included as dead loads.

#### 1.2.2 Live loads

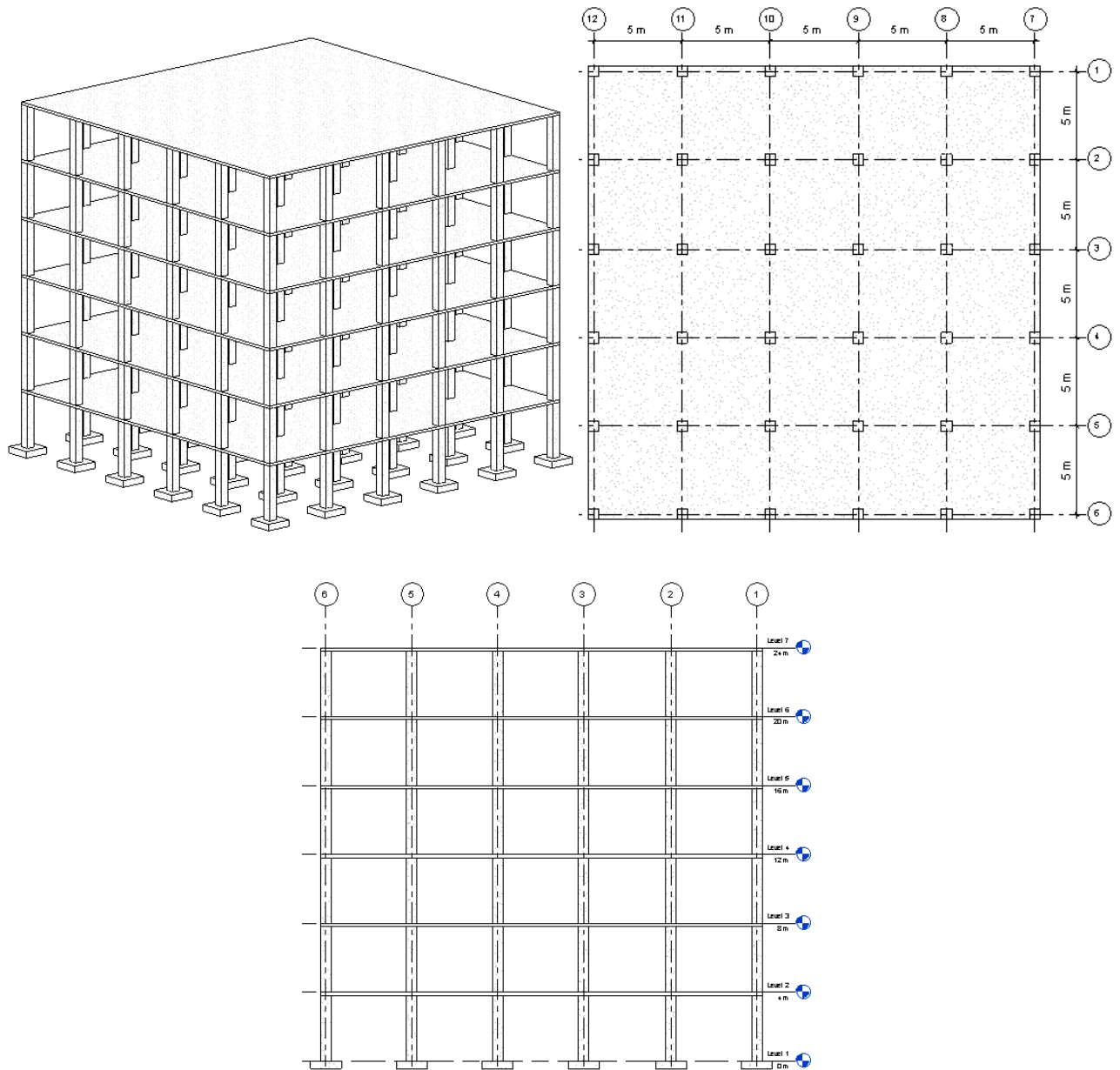
Consist chiefly of occupancy loads in buildings and traffic loads on bridges. They may be either fully or partially in place or not present at all, and may also change in location. Their magnitude and distribution at any given time are uncertain, and even their maximum intensities throughout the lifetime of the structures are not known with precision for values for live load to be used in building are found in **ASCE 7-10**.

#### 1.2.3 Environmental loads

Consist mainly of snow loads, wind pressure, earthquake loads, soil pressures on subsurface portions of structures, loads from possible ponding of rainwater on flat surface, and force caused by temperature differentials. Like live load  $s$ , environmental loads at any given time are uncertain in both magnitude and distribution.

**Example:** For the flat plate system shown below in figure below is proposed for a hospital building. Almost all floors to be patient rooms.

- According to requirements of ASCE 7-10, select an appropriate value for floor live load.



**Solution:**

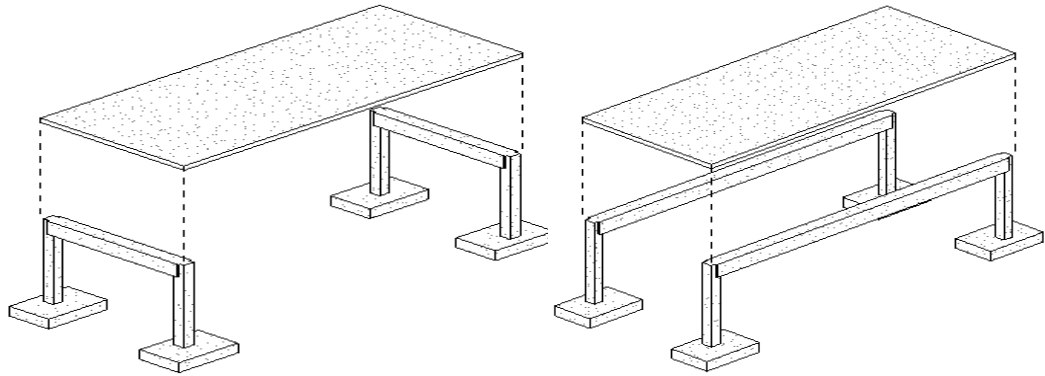
According to **Table 4-1** from **ASCE 7-10**  
 Minimum live ( $W_L$ ) load for hospitals for patient rooms is equal to  $1.92 \text{ kN/m}^2$  ■

Hospitals	
Operating rooms, laboratories	60 (2.87)
Patient rooms	40 (1.92)
Corridors above first floor	80 (3.83)

## 1.3 Types of slabs

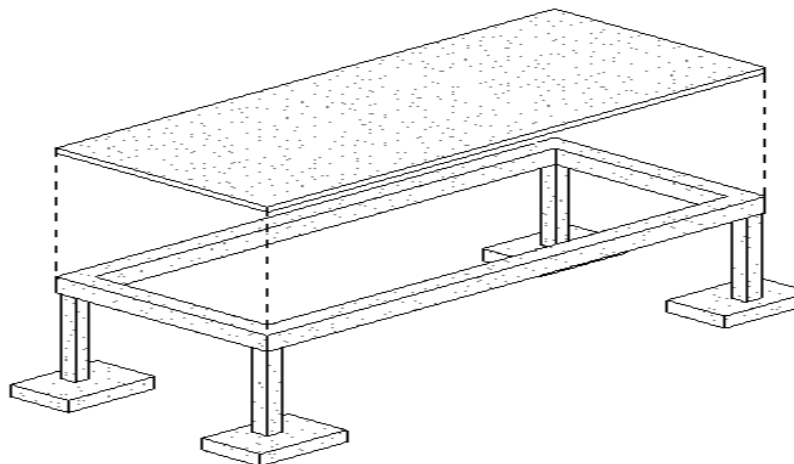
### 1.3.1 One Way Slabs

Slab may be supported on two opposite sides only as shown in **Fig. 1.1**, in which case the structural action of the slab is essentially one-way, the loads being carried by the slab in the direction perpendicular to the supporting beams the design of one-way slab was discussed in junior year.



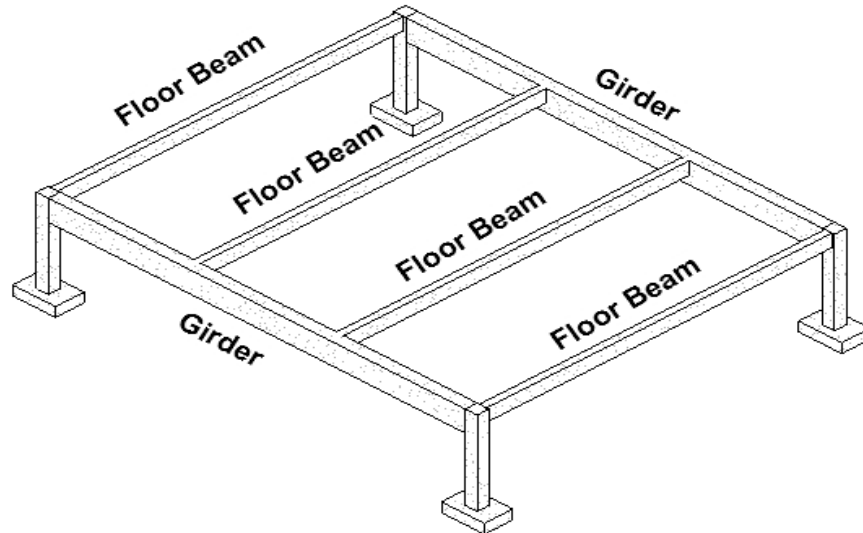
**Fig.1.1** One way slab supported on two opposite sides only.

There may be beams on all four sides, as shown in **Fig. 1.2** and of the ratio of length to width of one slab panel is larger than about 2, most of the load is carried in short direction to the supporting beams and **one-way action** is obtained in effect, **even though supports are provided on all sides**.



**Fig.1.2** One way slab supported on four sides with length to width larger than 2.

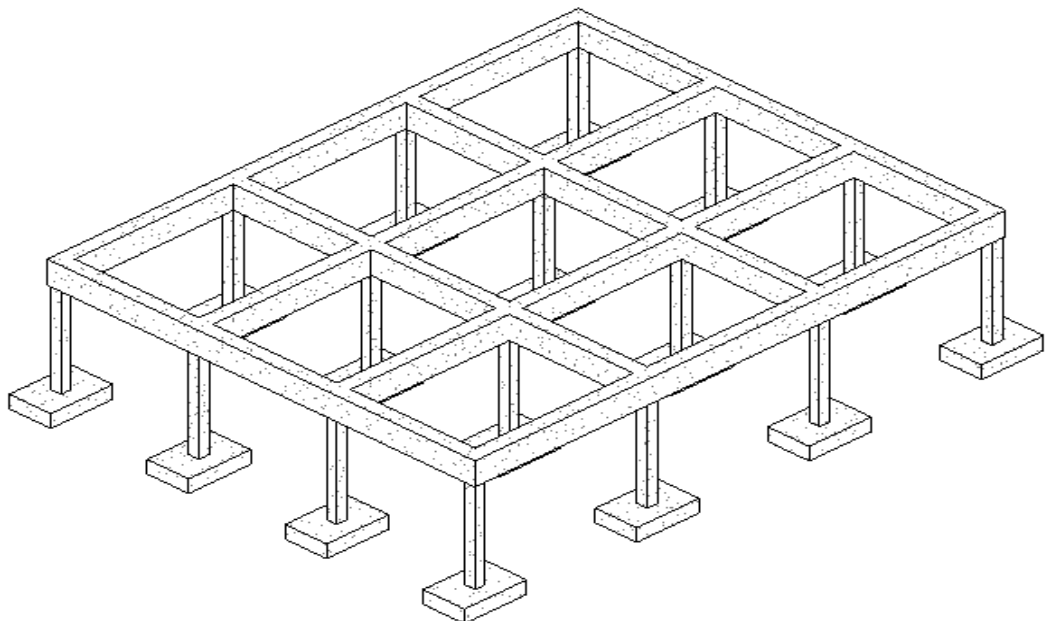
- For large column spacing, load may be transferred from the slab to the floor beams, then to larger beams (usually called the girders) and in turn to the supporting columns.



## 1.3.2 Two Way Slab

### 1.3.2.1 Two Way Slab with beams

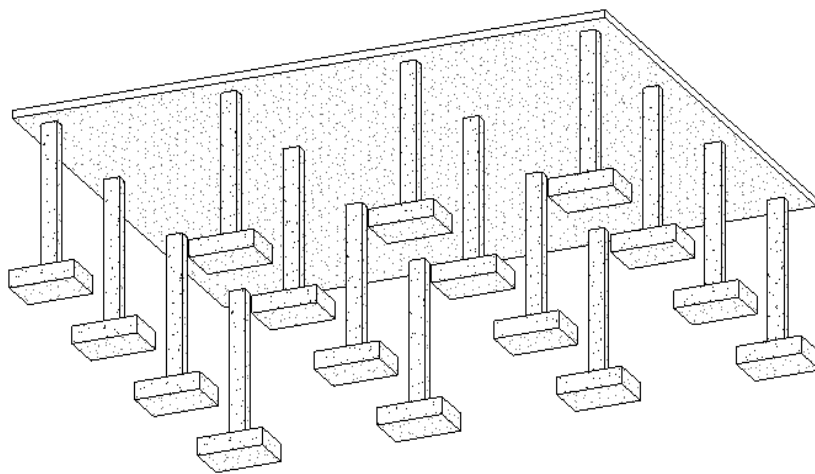
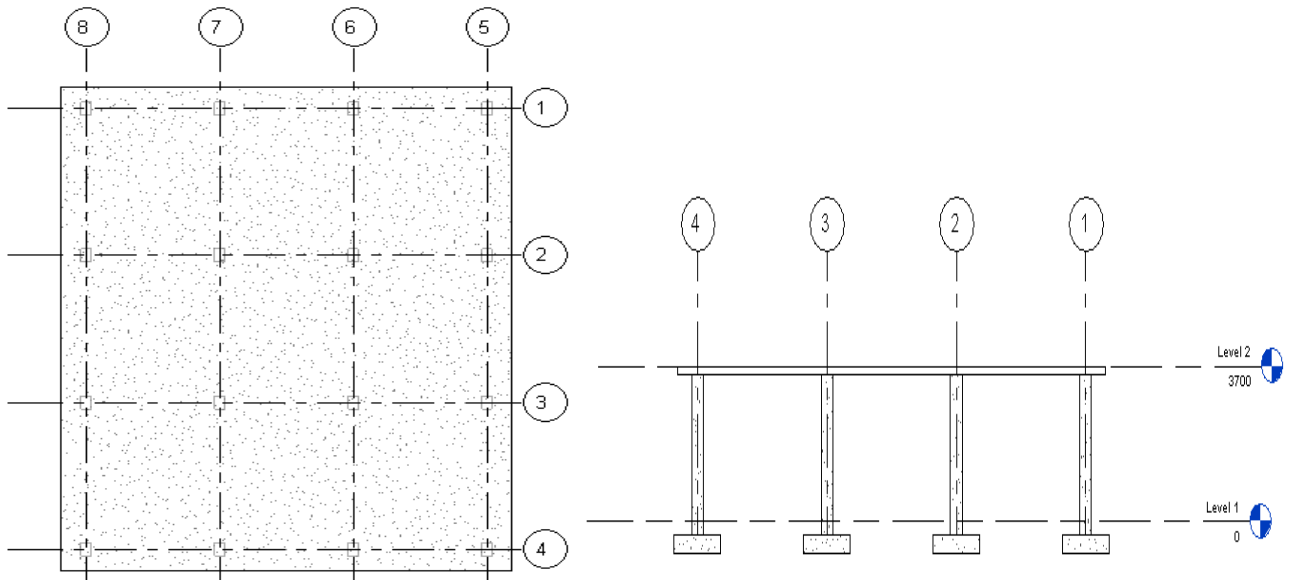
When slab supported on four sides with ratio of length to width equal or less than 2, so that two way slab action is obtained as shown in **Fig. 1.3**.



**Fig.1.3** two way slabs supported on four sides with length to width equal or less than 2.

### 1.3.2.1 Flat plate

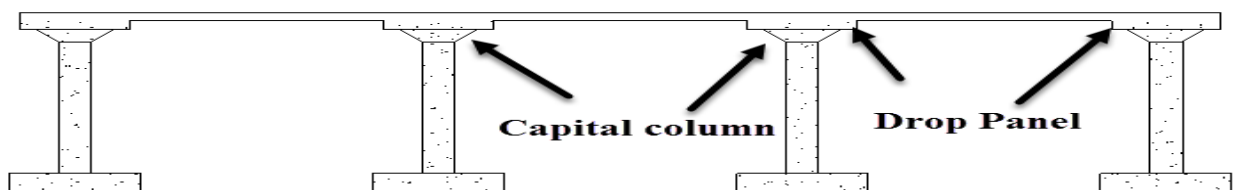
Concrete slabs in some cases may be carried directly by columns, as shown in **Fig. 1.4**. Without the use of beams or girders, such slabs are described as **flat plates**, this type of slab maybe used when the spans are not large and loads particularly not heavy.



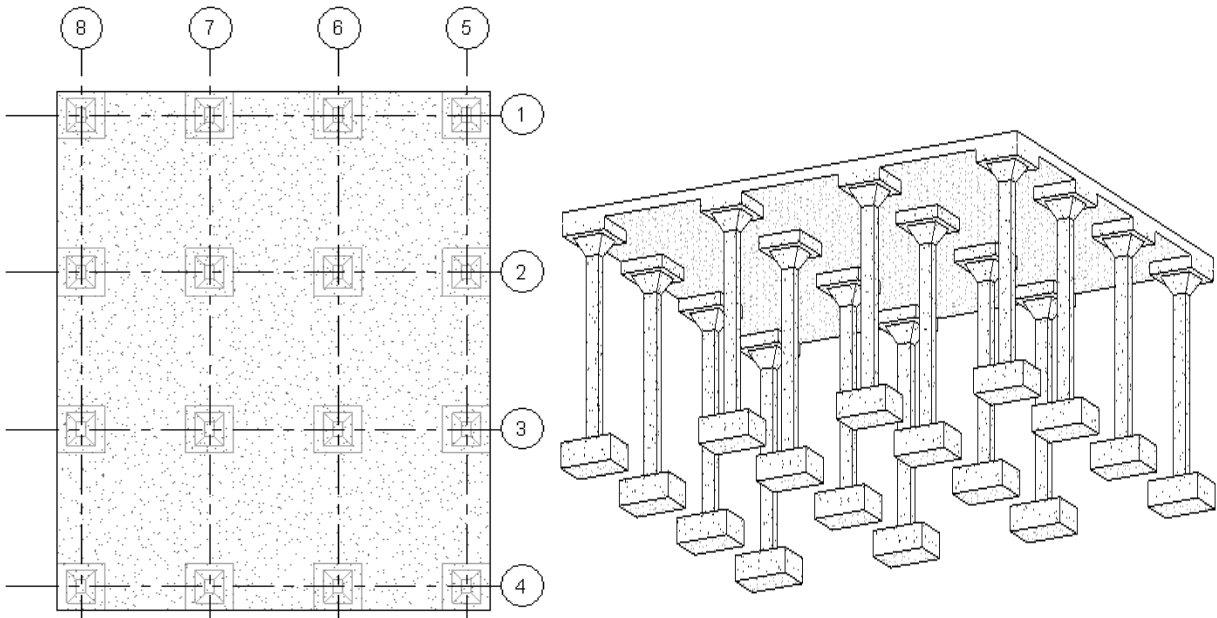
**Fig.1.4** two way flat plates.

### 1.3.2.2 Flat Slab

Flat slab is also beamless but incorporates a thickened slab region in the vicinity of the column and often employs flared column tops. Both are devices to reduce the stresses due to negative bending and shear around the columns, they are referred as **drop panels** and **column capitals** respectively. **Fig. 1.5a and b** is showing the flat slab.

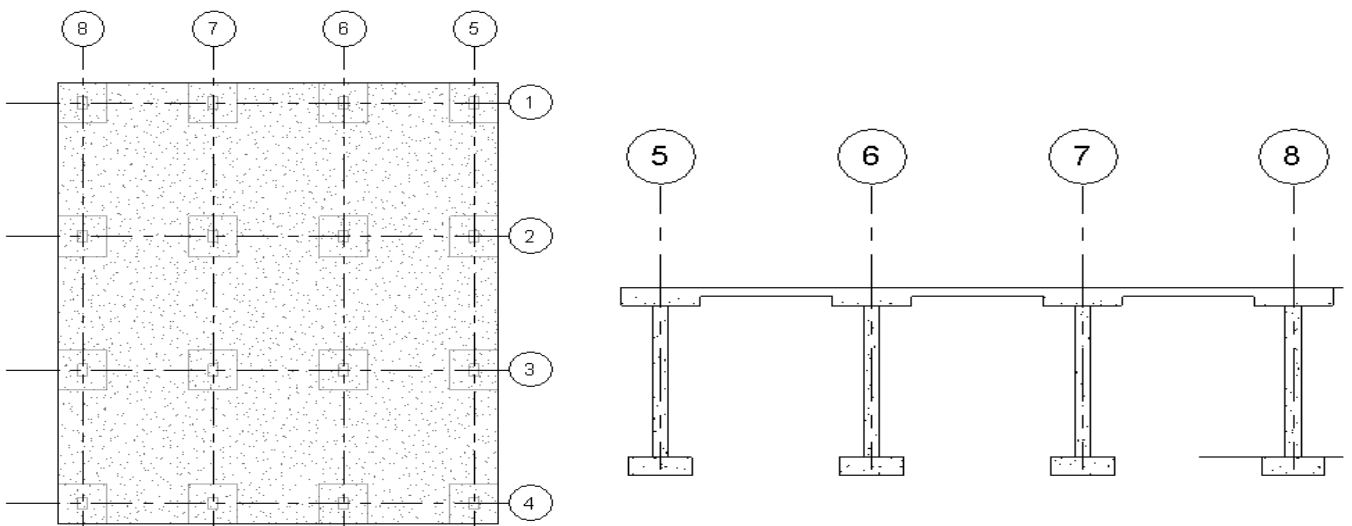


**Fig.1.5a** two way flat slabs with both drop panels and column capitals.



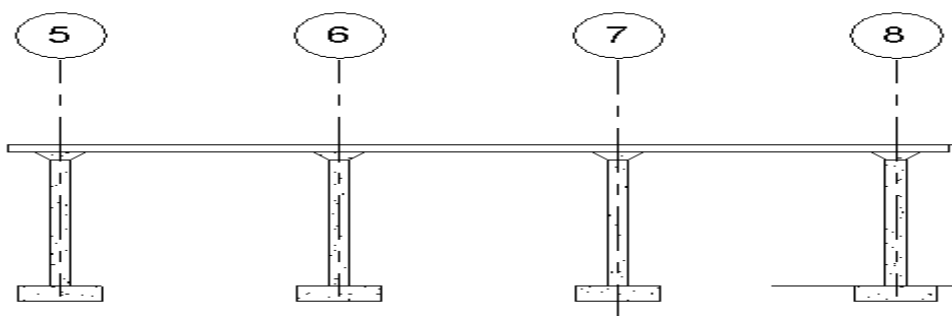
**Fig.1.5b** two way flat slabs with both drop panels and column capitals.

- Slabs may have only drop panels, this type of slab commonly used in parking garage as shown in **Fig. 1.6**.



**Fig.1.6** two way flat slabs drop panels only.

- Slab may be constructed with capital column without drop panel as shown in **Fig1.7**; this type of slab is rarely.



**Fig.1.7** two way flat slabs column capital only.

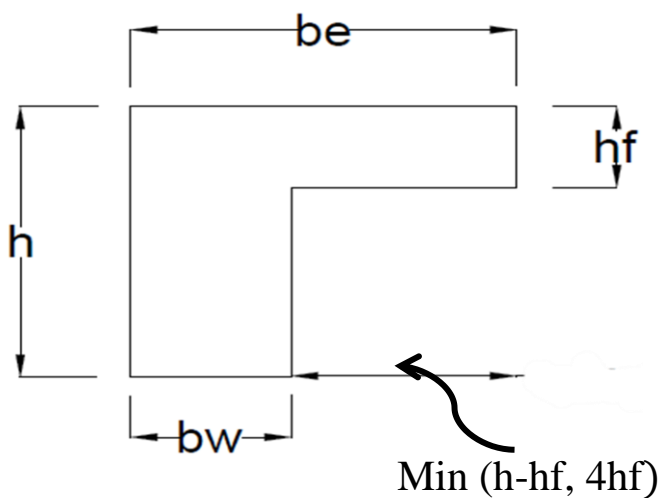
- Very often slabs built **without interior beams** between the columns but the **edge beams** running around **the perimeter** of the building as shown in **Fig. 1.8**. These beams are very helpful in **stiffening** the slabs and **reducing the deflections** in the exterior slab panels.
- The stiffness of slabs with edge beams is expressed as a function of  $\alpha_f$ . This expression is used to represent the **ration of flexural stiffness of a beam (  $E_{cb} \cdot I_b$  ) to flexural stiffness of the slab (  $E_{cs} \cdot I_s$  )**. If no beams are used as in the case for **the flat plate  $\alpha_f$  will equal to 0.**

$$\alpha_f = \frac{E_{cb} \cdot I_b}{E_{cs} \cdot I_s} = \frac{E_{cb} \cdot I_b}{E_{cs} \cdot I_s} = \frac{I_b}{I_s}$$

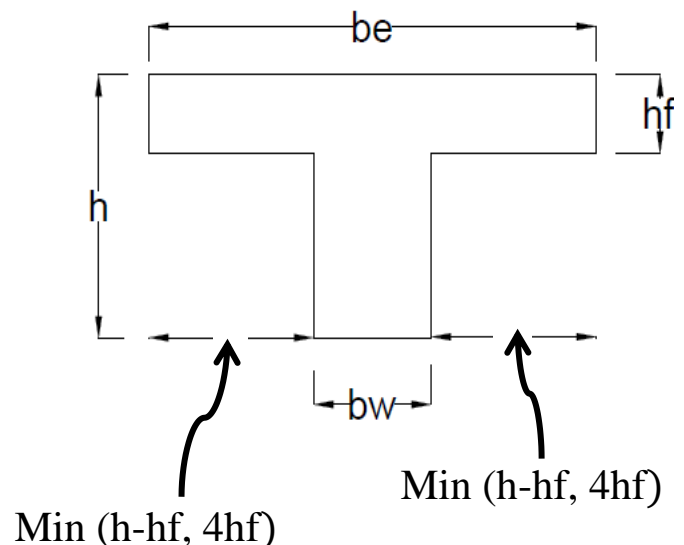
- In which  $E_{cb}$  and  $E_{cs}$  are the moduli of elasticity of the beam and slab concrete (**usually the same**) and  $I_b$  and  $I_s$  are the moments of inertia of the effective beam and the slab.
- The moment of inertia of a flange beam about it is own centroid axis can be computed based on simple definition of centroid or by approximate method.

$$I_b = k \cdot \frac{b_w \cdot h^3}{12}$$

$$k \approx 1 + 0.2 \left( \frac{b_e}{b_w} \right) \quad \text{for} \quad 2 < \frac{b_e}{b_w} < 4 \quad \text{and} \quad 0.2 < \frac{h_f}{h} < 0.5$$



L-beam (Exterior Beam)

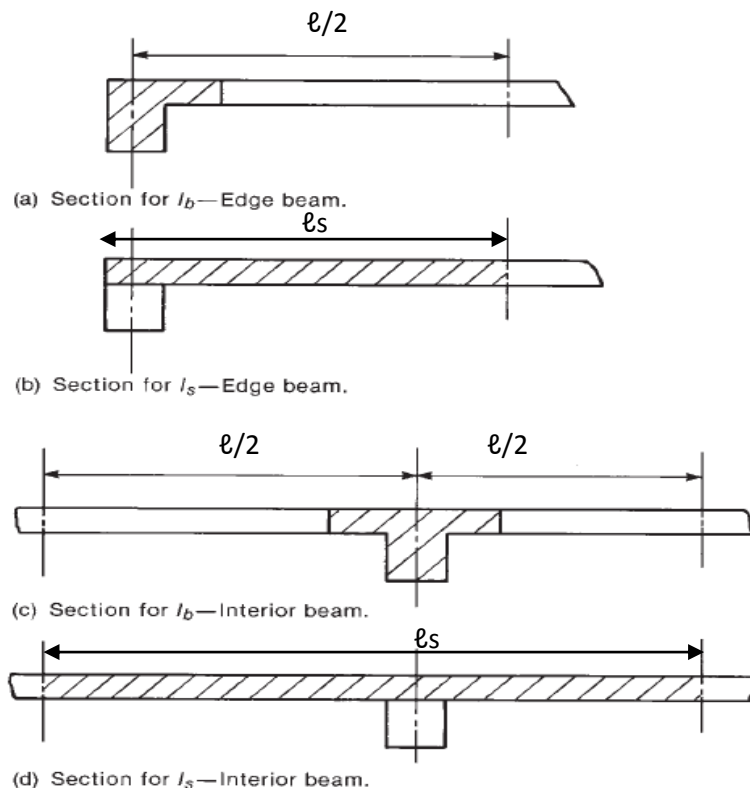


T-beam (interior Beam)

- $I_s = \frac{\ell_s \cdot h_{slab}^3}{12}$

$\ell_s$  = is the width of the frame in the direction **perpendicular** to the beam required to calculate  $\alpha_f$ .

$h_{slab}$  = thickness of slab



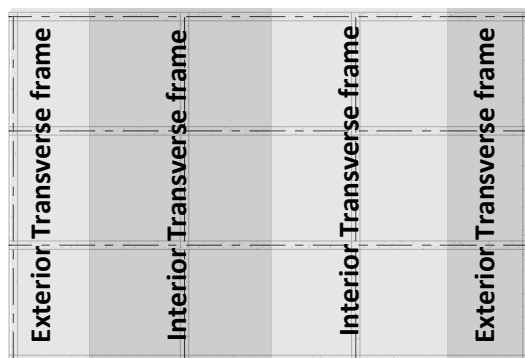
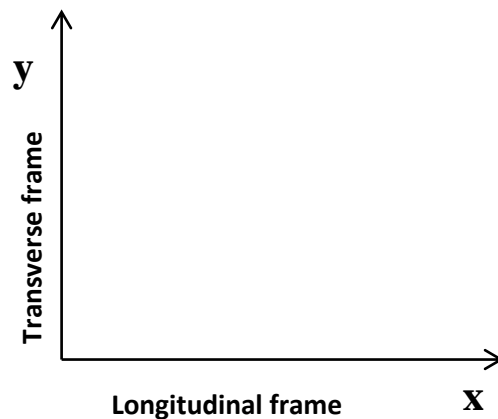
## Types of frames in slab

### 1. Longitudinal frames

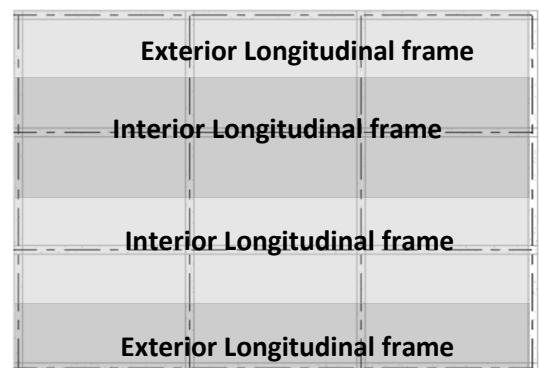
- Exterior frame
- Interior frame

### 2. Transverse frames

- exterior frame
- Interior frame



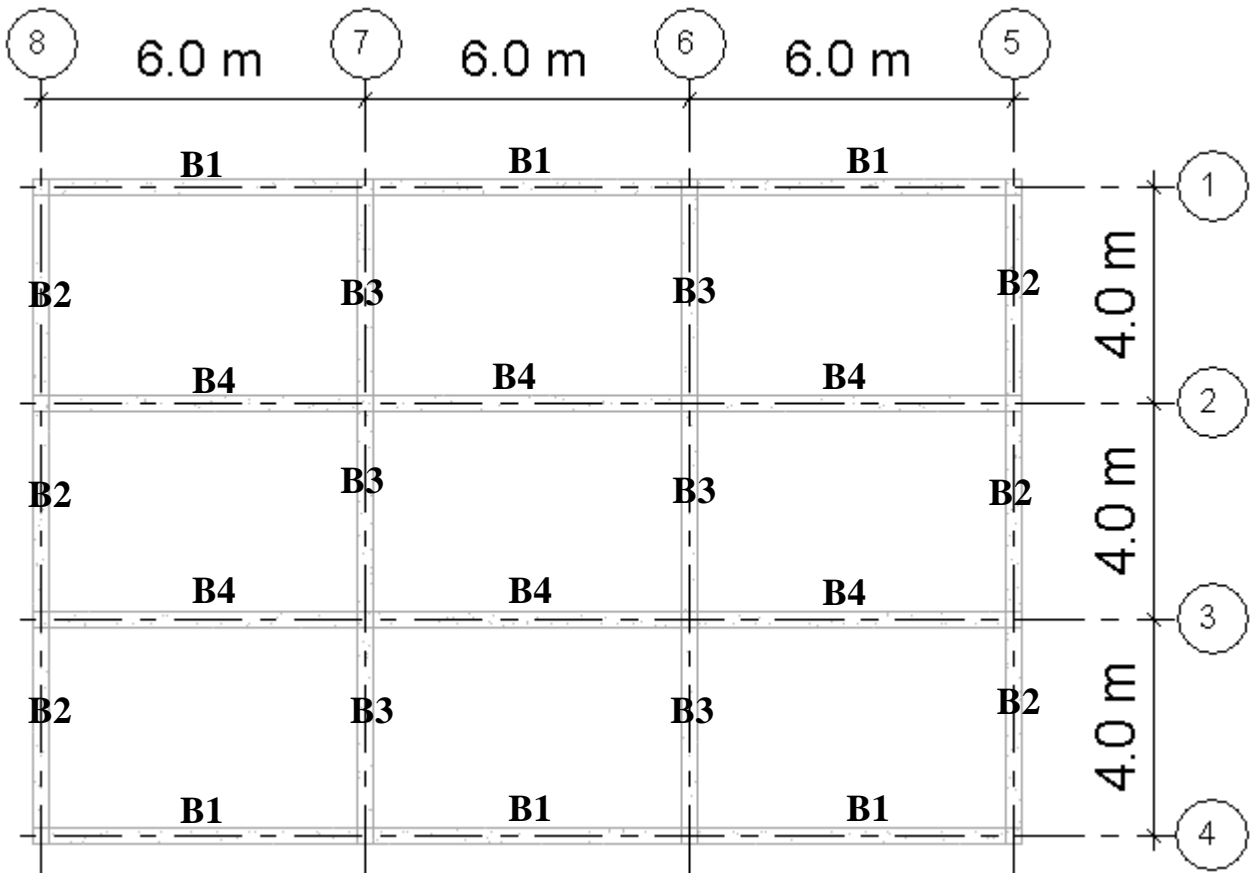
Transverse frames



Longitudinal frames



**Example:** for the slab showing below calculate  $\alpha_f$  for B1, B2, B3 and B4  
 All beams dimensions are (300 x 600) and columns dimensions (300 x 300) mm and slab thickness is (180) mm.



**Solution:**

**For B1**

$\text{Min}(h-h_f, 4h_f) = \text{min}(600-180, 4*180)$

$\text{Min}(420, 720)$  use **420** mm

$b_e = 300 + 420 = 720$

$\alpha_f = \frac{I_b}{I_s}$

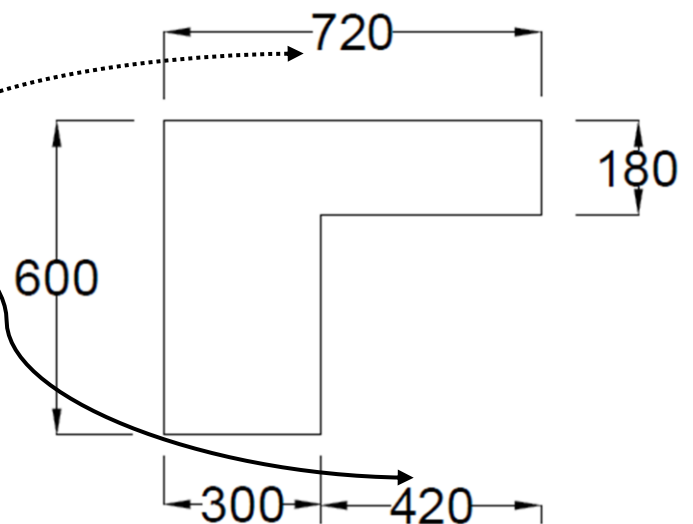
$I_b = k * \frac{b_w \cdot h^3}{12}$

Check

$2 < \frac{b_e}{b_w} = \frac{720}{300} = 2.4 < 4$

$0.2 < \frac{h_f}{h} = \frac{180}{600} = 0.3 < 0.5$  } O.k

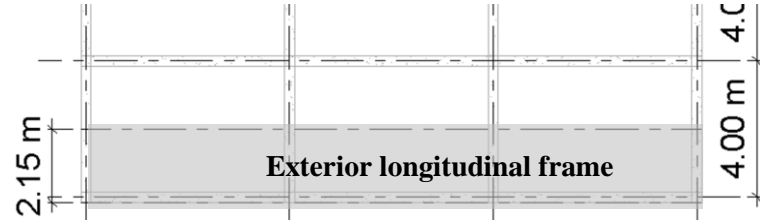
$k = 1 + 0.2 \left( \frac{b_e}{b_w} \right) = 1 + 0.2 * 2.4 = 1.48$



Section for B1 (exterior beam)

$$I_b = k * \frac{b_w \cdot h^3}{12} = 1.48 * \frac{300 * 600^3}{12} = 7.992 * 10^9 \text{ mm}^4$$

$$\ell_s = \frac{\ell_2}{2} + \frac{\text{Column width}}{2} = \frac{4000}{2} + \frac{300}{2} = 2150 \text{ mm}$$



$$I_s = \frac{\ell_s * h_{slab}^3}{12} = \frac{2150 * 180^3}{12} = 1.045 * 10^9 \text{ mm}^4$$

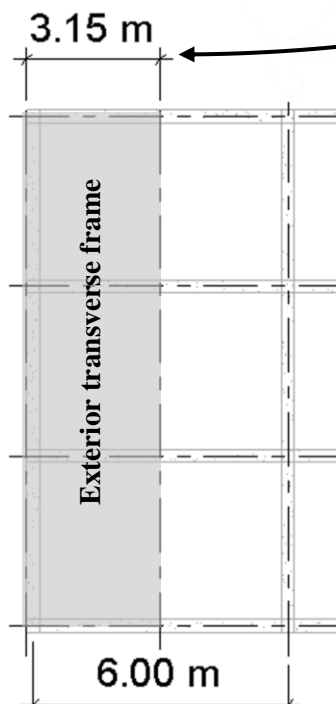
$$\alpha_{f1} = \frac{I_b}{I_s} = \frac{7.992 * 10^9}{1.045 * 10^9} = 7.64$$

### For B2

Beam (B2) has the same moment of inertia of (B1) **because the beam has the same dimensions.**

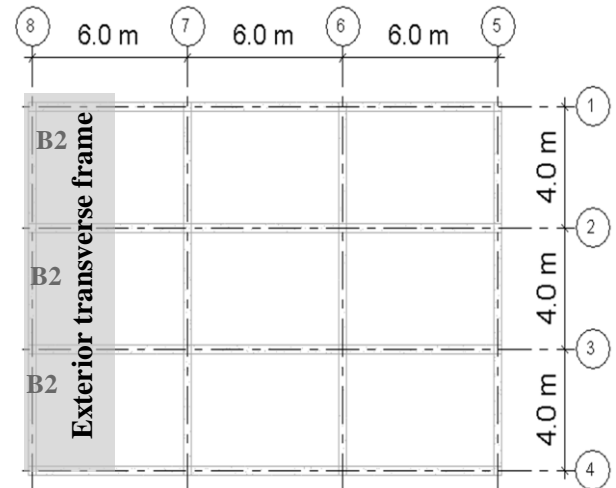
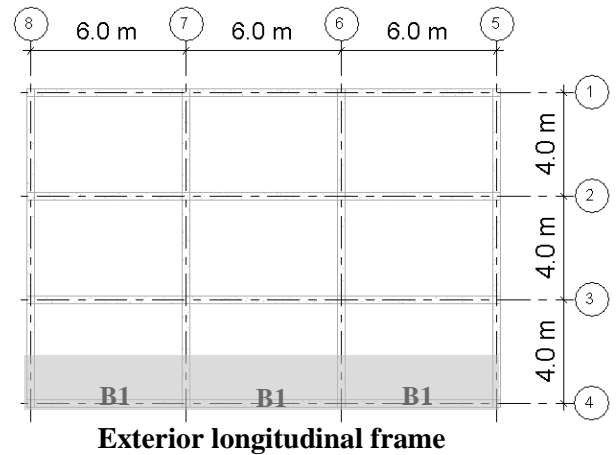
$$I_b = k * \frac{b_w \cdot h^3}{12} = 1.48 * \frac{300 * 600^3}{12} = 7.992 * 10^9 \text{ mm}^4$$

$$\ell_s = \frac{\ell_2}{2} + \frac{\text{Column width}}{2} = \frac{6000}{2} + \frac{300}{2} = 3150 \text{ mm}$$



$$I_s = \frac{\ell_s * h_{slab}^3}{12} = \frac{3150 * 180^3}{12} = 1.53 * 10^9 \text{ mm}^4$$

$$\alpha_{f2} = \frac{I_b}{I_s} = \frac{7.992 * 10^9}{1.53 * 10^9} = 5.223$$



## For B3

$$\text{Min}(h-h_f, 4h_f) = \text{min}(600-180, 4*180)$$

$$\text{Min}(420, 720) \text{ use } \mathbf{420} \text{ mm}$$

$$b_e = 420 + 300 + 420 = \mathbf{1140} \text{ mm}$$

$$\alpha_f = \frac{I_b}{I_s}$$

$$I_b = k * \frac{b_w \cdot h^3}{12}$$

Check

$$2 < \frac{b_e}{b_w} = \frac{1140}{300} = 3.8 < 4$$

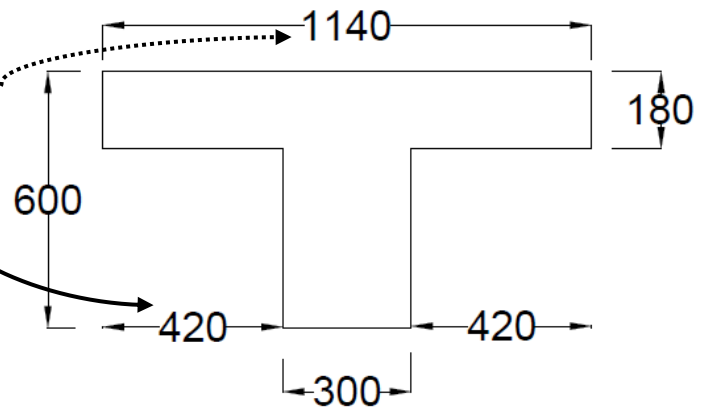
$$0.2 < \frac{h_f}{h} = \frac{180}{600} = 0.3 < 0.5$$

O.k

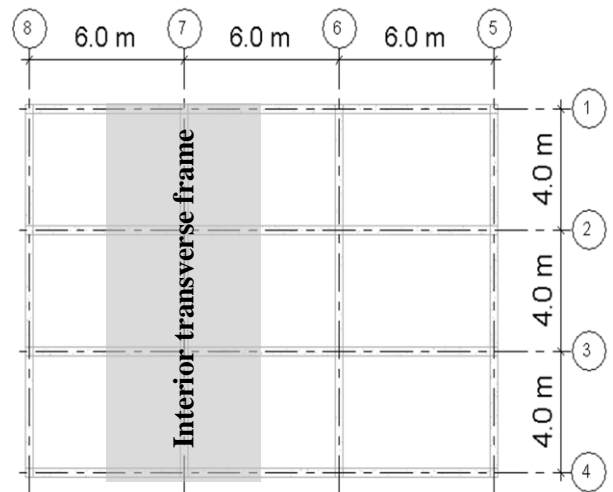
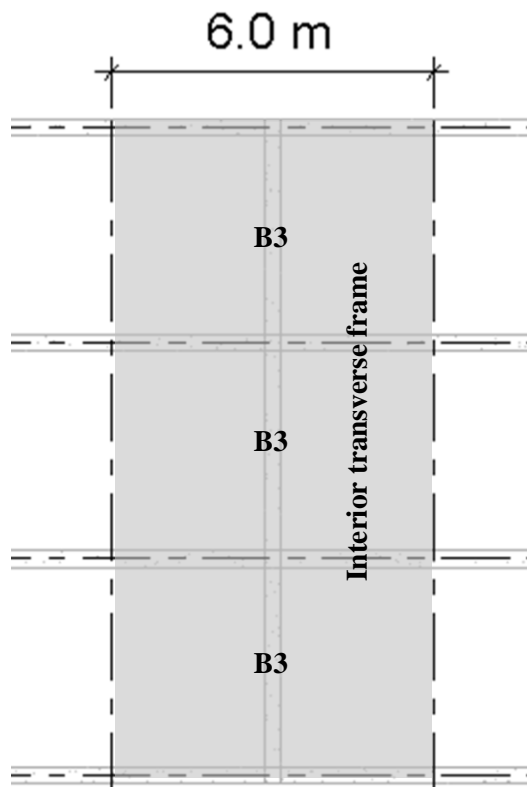
$$k = 1 + 0.2 \left( \frac{b_e}{b_w} \right) = 1 + 0.2 * 3.8 = 1.76$$

$$I_b = k * \frac{b_w \cdot h^3}{12} = 1.76 * \frac{300 * 600^3}{12} = 9.504 * 10^9 \text{ mm}^4$$

$$\ell_s = \frac{\ell_1}{2} + \frac{\ell_1}{2} = \frac{6000}{2} + \frac{6000}{2} = 6000 \text{ mm}$$



Section for B3 (interior beam)



$$I_s = \frac{\ell_s * h_{slab}^3}{12} = \frac{6000 * 180^3}{12} = 2.916 * 10^9 \text{ mm}^4$$

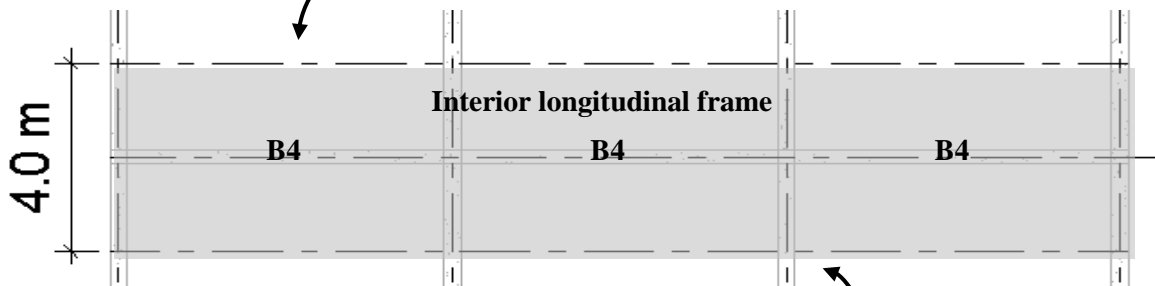
$$\alpha_{f3} = \frac{I_b}{I_s} = \frac{9.504 * 10^9}{2.916 * 10^9} = 3.259$$

## For B4

Beam (B4) has the same moment of inertia of (B3) because the beam has the same dimensions.

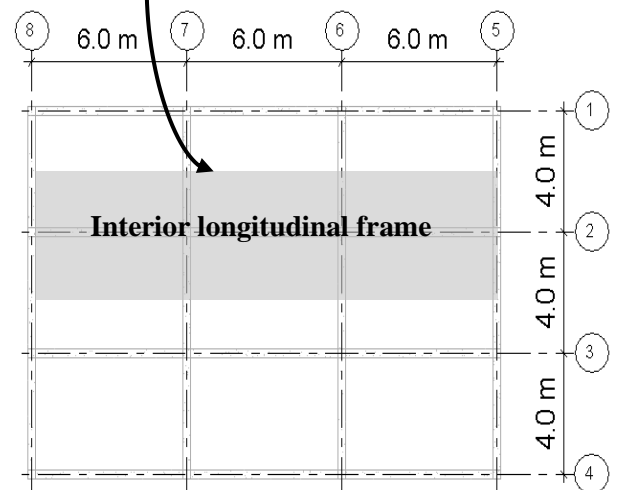
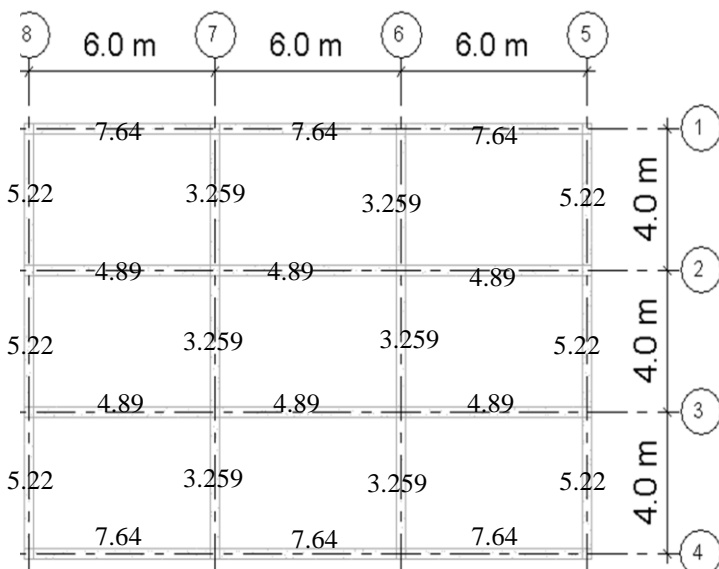
$$I_b = k * \frac{b_w \cdot h^3}{12} = 1.76 * \frac{300 * 600^3}{12} = 9.504 * 10^9 \text{ mm}^4$$

$$\ell_s = \frac{\ell_2}{2} + \frac{\ell_2}{2} = \frac{4000}{2} + \frac{4000}{2} = 4000 \text{ mm}$$

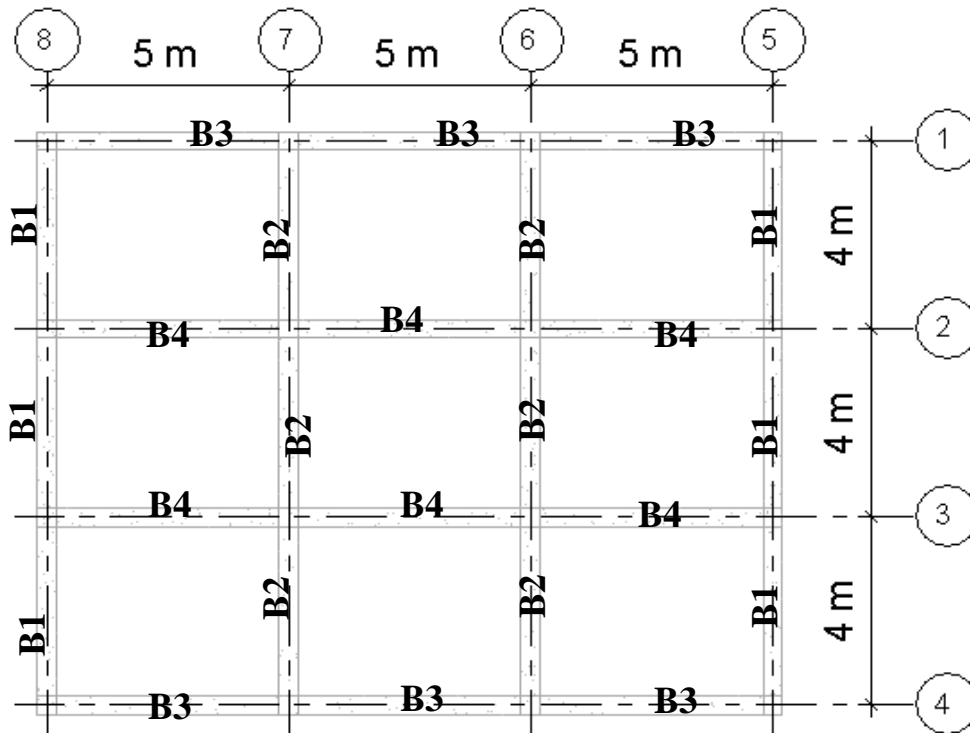


$$I_s = \frac{\ell_s * h_{slab}^3}{12} = \frac{4000 * 180^3}{12} = 1.944 * 10^9 \text{ mm}^4$$

$$\alpha_{f4} = \frac{I_b}{I_s} = \frac{9.504 * 10^9}{1.944 * 10^9} = 4.89$$



**Example:** For the slab with beams shown below calculate  $\alpha_f$  for **B1, B2, B3** and **B4**. All beams and columns dimensions are (600 x 400) mm (400 x 400) mm respectively, and slab thickness is (150) mm



**Solution:**

**Calculating  $\alpha_f$  for B1**

$$\alpha_f = \frac{I_b}{I_s}$$

Check

$$2 < \frac{b_e}{b_w} = \frac{850}{400} = 2.125 < 4$$

$$0.2 < \frac{h_f}{h} = \frac{150}{600} = 0.25 < 0.5$$

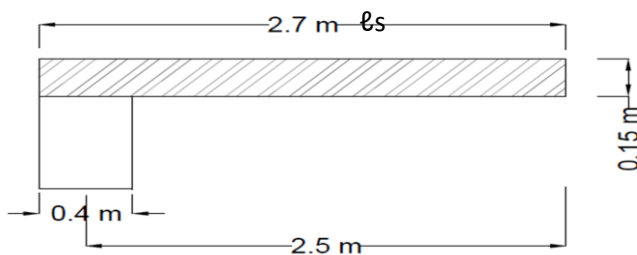
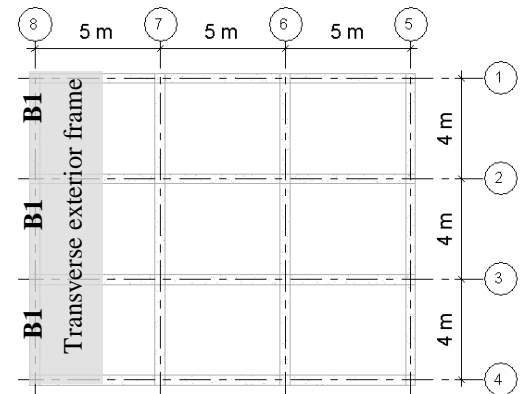
$$k = 1 + 0.2 \left( \frac{b_e}{b_w} \right) = 1 + 0.2 * 2.125 = 1.425$$

$$I_b = k * \frac{b_w * h^3}{12} = 1.425 * \frac{400 * 600^3}{12} = 1.026 * 10^{10}$$

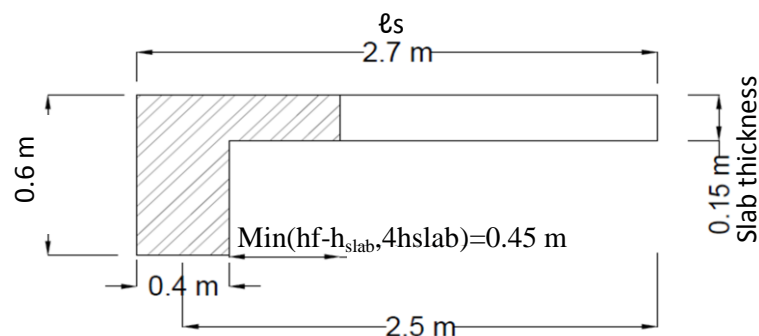
$$\ell_s = 2500 + 200 = 2700 \text{ mm}$$

$$I_s = \frac{\ell_s * h_{slab}^3}{12} = \frac{2700 * 150^3}{12} = 7.59 * 10^8 \text{ mm}^4$$

$$\alpha_{f1} = \frac{I_b}{I_s} = \frac{1.026 * 10^{10}}{7.59 * 10^8} = 13.52$$



Slab section for B1 transverse exterior frame



Beam section for B1 in transverse exterior frame

### Calculating $\alpha_f$ for B2

$$\alpha_f = \frac{I_b}{I_s}$$

Check

$$2 < \frac{b_e}{b_w} = \frac{1300}{400} = 3.25 < 4$$

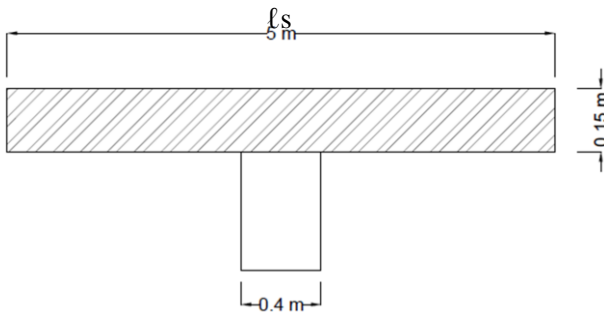
$$0.2 < \frac{h_f}{h} = \frac{150}{600} = 0.25 < 0.5$$

$$k = 1 + 0.2 \left( \frac{b_e}{b_w} \right) = 1 + 0.2 * 3.25 = 1.65$$

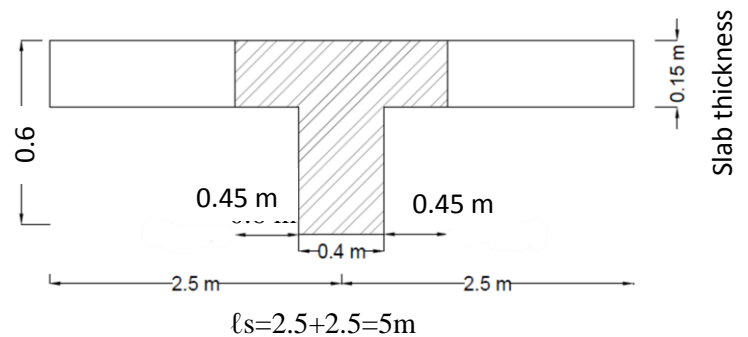
$$I_b = k * \frac{b_w * h^3}{12} = 1.65 * \frac{400 * 600^3}{12} = 1.188 * 10^{10}$$

$$I_s = \frac{\ell_s * h_{slab}^3}{12} = \frac{5000 * 150^3}{12} = 1.4 * 10^9 \text{ mm}^4$$

$$\alpha_{f2} = \frac{I_b}{I_s} = \frac{1.188 * 10^{10}}{1.4 * 10^9} = 8.48$$



Slab section for B2 in transverse interior frame



Beam section for B2 in transverse interior frame

### Calculating $\alpha_f$ for B3

$$\alpha_f = \frac{I_b}{I_s}$$

$$I_b = 1.026 * 10^{10}$$

$$\ell_s = 2000 + 200 = 2200 \text{ mm}$$

$$I_s = \frac{\ell_s * h_{slab}^3}{12} = \frac{2200 * 150^3}{12} = 6.18 * 10^8 \text{ mm}^4$$

$$\alpha_{f3} = \frac{I_b}{I_s} = \frac{1.026 * 10^{10}}{6.18 * 10^8} = 16.6$$

### Calculating $\alpha_f$ for B4

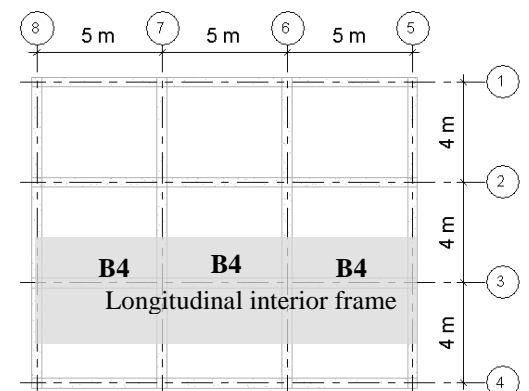
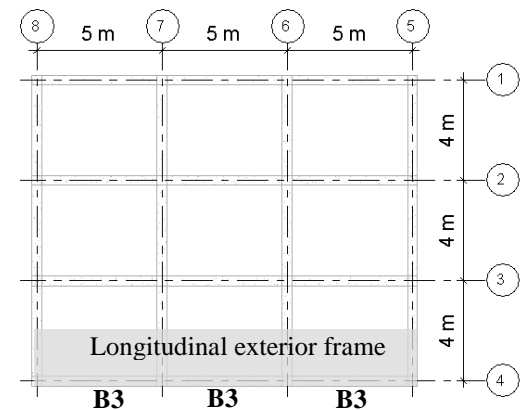
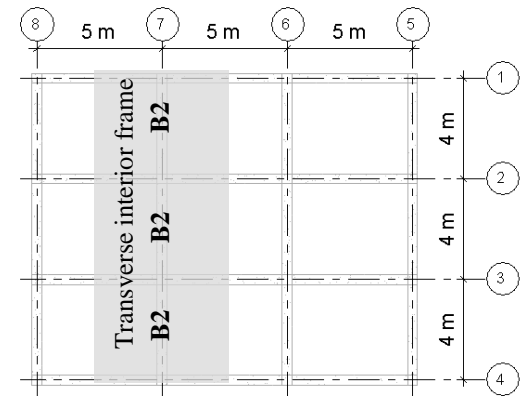
$$\alpha_f = \frac{I_b}{I_s}$$

$$I_b = 1.188 * 10^{10}$$

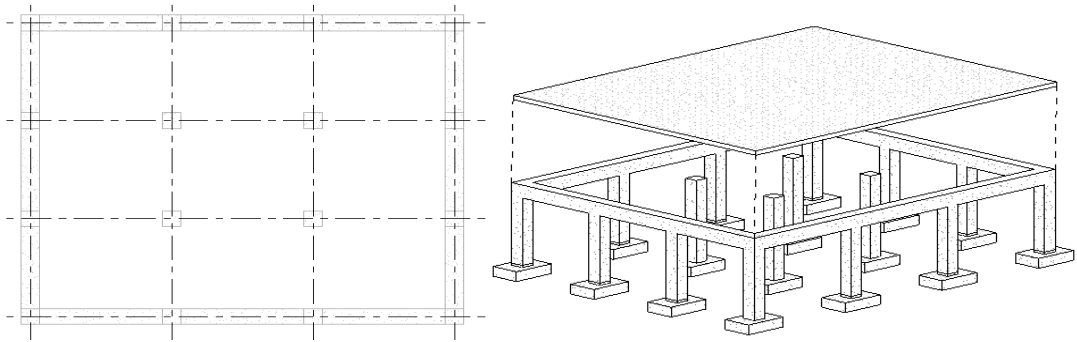
$$\ell_s = 2000 + 2000 = 4000 \text{ mm}$$

$$I_s = \frac{\ell_s * h_{slab}^3}{12} = \frac{4000 * 150^3}{12} = 1.125 * 10^9 \text{ mm}^4$$

$$\alpha_{f4} = \frac{I_b}{I_s} = \frac{1.188 * 10^{10}}{1.125 * 10^9} = 10.56 \quad \blacksquare$$

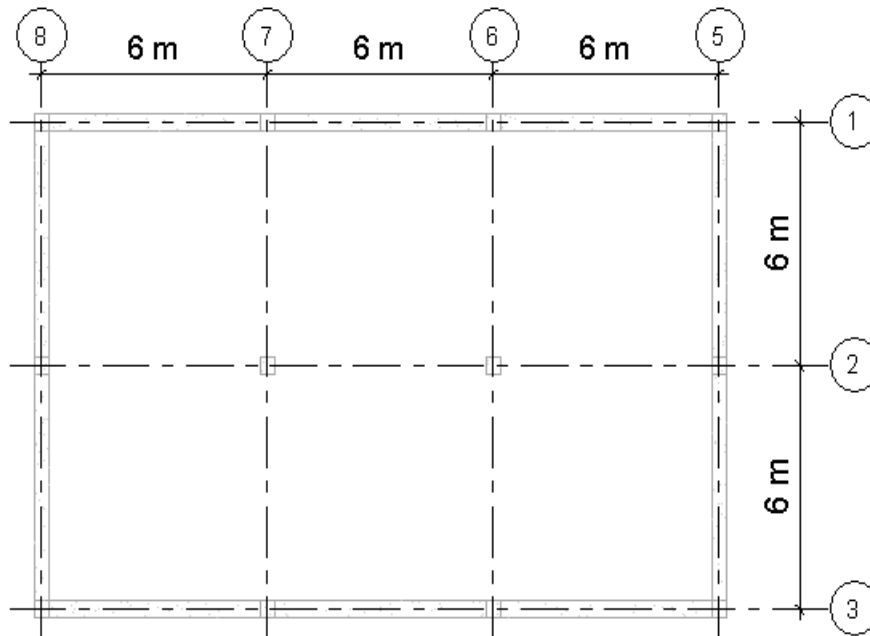


- For slabs with beams between columns along **exterior edges**  $\alpha_f$  for the edge beams **may not be < 0.8**.



**Fig.1.8** two way flat slabs with edge beams.

**Example:** Check whether if the slab below is considered as a slab with or without edge beam according to ACI requirements. Edge beams dimensions are (600x250) mm and slab thickness is (150) mm, Column dimensions (250 x 250) mm



**Solution:**

- According to footnote in **table 8.3.1.1 of ACI Code** for slabs with beams between columns along exterior edge  $\alpha_f$  for the edge beams shall **not be less than 0.8**, otherwise the slab will be considered as slab without edge beams.

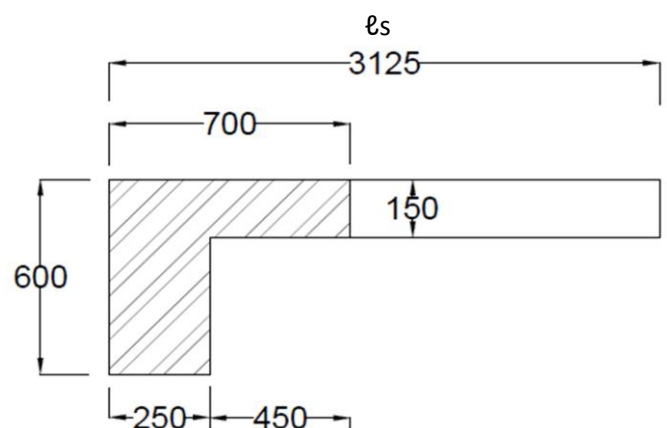
$$\alpha_f = \frac{l_b}{l_s}$$

Check

$$2 < \frac{b_e}{b_w} = \frac{700}{250} = 2.8 < 4$$

$$0.2 < \frac{h_f}{h} = \frac{150}{600} = 0.25 < 0.5$$

$$k = 1 + 0.2 \left( \frac{b_e}{b_w} \right) = 1 + 0.2 * 2.8 = 1.56$$



$$I_b = k * \frac{b_w \cdot h^3}{12} = 1.56 * \frac{250 * 600^3}{12} = 7.02 * 10^9 \text{ mm}^4$$

$$l_s = 3000 + 125 = 3125 \text{ mm}$$

$$I_s = \frac{l_s * h_{slab}^3}{12} = \frac{3125 * 150^3}{12} = 8.78 * 10^8$$

$$\alpha_f = \frac{I_b}{I_s} = \frac{7.02 * 10^9}{8.78 * 10^8} = 7.995 > 0.8$$

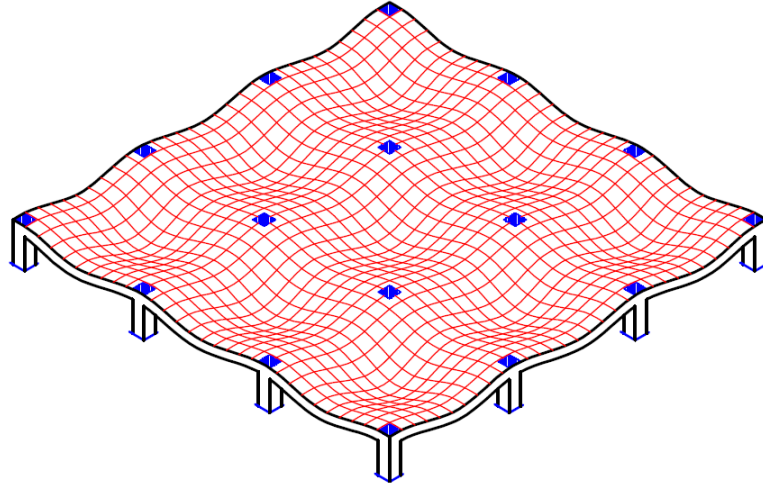
Then the slab is considered with edge beams. ■



## 1.4 Behavior of two way slab

Two-way slabs bend under load into dish-shaped surface, so there is bending in both principal directions. As a result they must be reinforcement in both directions by layers of bars that are perpendicular to each other.

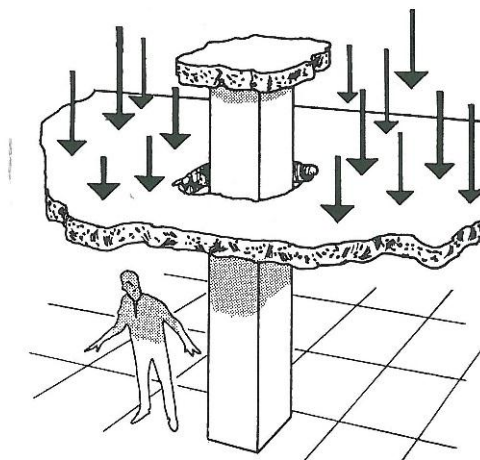
The ACI Code specifies two methods for designing two-way slabs. These are the **direct design method** and **the equivalent frame method**.



- The direct-design method is emphasized in this course because an understanding of the method is essential for understanding the concepts of two-way slab design. In addition, it is an excellent method of checking slab design calculations.

## 1.5 Minimum thickness for Drop panel

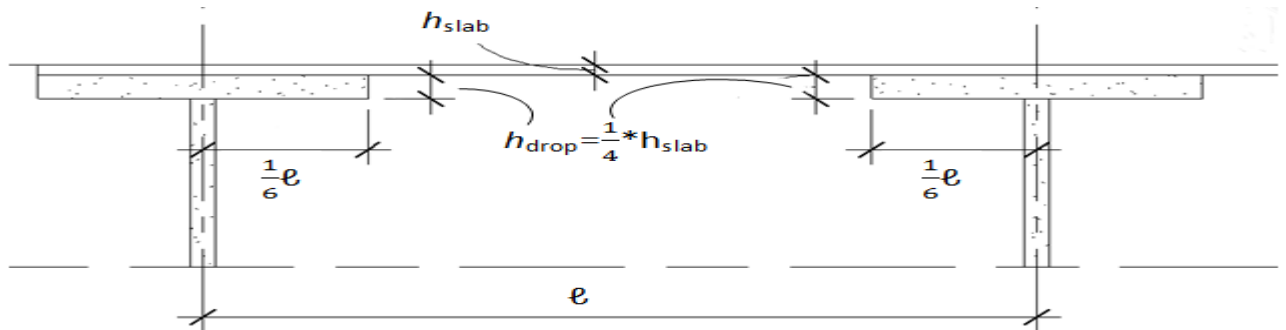
Flat plates present a possible problem in transferring the shear at the perimeter of the columns. In other words, there is a danger that the columns may punch through the slabs as shown in **Fig 1.9**. Then the slab can be strengthened by thickening of the slabs around the columns (**drop panels**) as shown in **Fig 1.6**.



**Fig 1.9** punching of two-way flat slab.

They are provided for three main reasons:

1. The minimum thickness of slab required to limit deflections may be reduced by 10% if the slab has drop panels, the drop panel stiffens the slab in the region of highest moments and hence reduces the deflection.
  2. A drop panel can be used to reduce the amount of negative-moment reinforcement required over a column in flat slab.
  3. A drop panel gives additional slab depth at the column, thereby increasing the area of the critical shear perimeter.
- According to **ACI Code 8.2.4** a drop panel shall project below the slab at least **one-fourth** of the slab thickness.
  - And shall extend in each direction from centerline of support a distance not less than **one-sixth** the span length measured from center to center of supports in that direction as shown in **Fig1.10**.

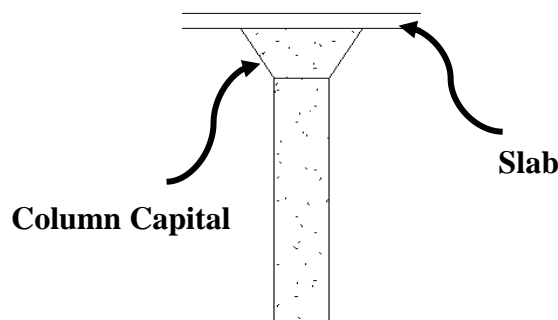


**Fig 1.10** Dimensions of Drop panel.

- If drop panels do not satisfy the length requirements given in **ACI Code 8.2.4** still can be used for added shear strength and are sometimes referred as **shear capitals** or **shear caps** (**ACI Code 8.2.5**).

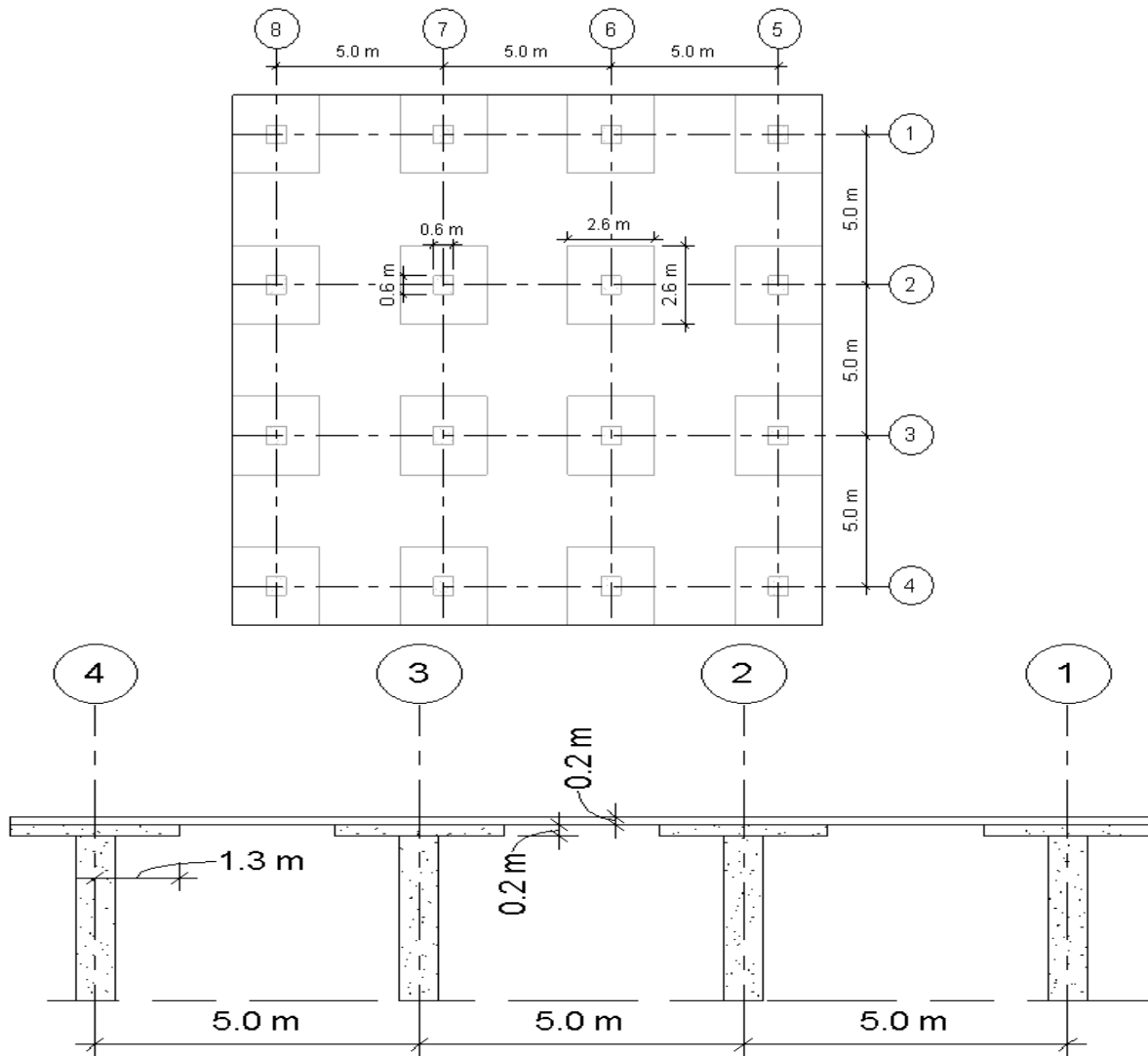
## 1.6 Column Capitals

Occasionally, the top of a column will be flared outward as shown below, then this called a column capital and it located directly below the slab or a drop panel that is cast **monolithically** with the column.



This is done to provide a larger shear perimeter at the column and to **reduce the clear span  $l_n$** , used in computing the moments.

**Example:** check the dimensions of drop panel shown in figure below according to ACI code. Column dimension is (600 x 600) mm, slab thickness is (200) mm, drop panel thickness is (400) mm.



**Solution:**

- Check the projected thickness for drop panel

According to ACI Code 8.2.4 a drop panel shall be project below the slab at least one-fourth of the slab thickness.

$$h_{drop} = 200 \text{ mm} \geq \frac{h_{slab}}{4}$$

$$h_{drop} = 200 \text{ mm} \geq \frac{200}{4}$$

$h_{drop} = 200 \text{ mm} > 50 \text{ mm}$  the projected thickness is ok.

- Check the length of drop panel

ACI Code state that each direction from centerline of support shall extend a distance not less than one-sixth the span length measured from center to center of supports in that direction

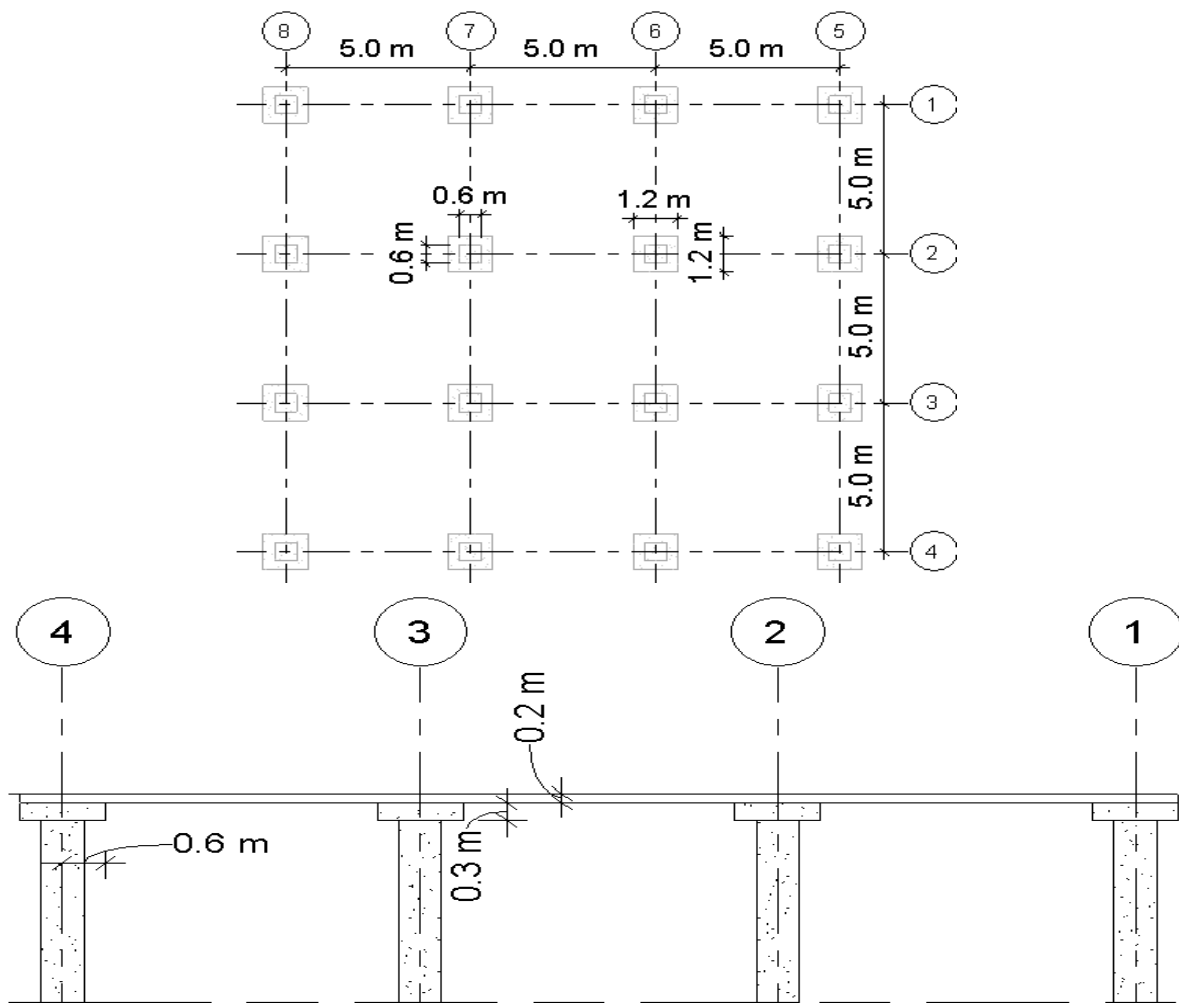
$$1.3 \geq \frac{l}{6}$$

$$1.3 \geq \frac{5}{6}$$

$$1.3 \text{ m} > 0.83 \text{ m ok}$$

All requirements are satisfying for a drop panel ■

**Example:** check the dimensions of drop panel shown in figure below according to ACI code. Column dimension is (600 x 600) mm, slab thickness is (200) mm, drop panel thickness is (500) mm.



**Solution:**

- Check the projected thickness for drop panel

$$h_{\text{drop}} = 300 \text{ mm} ? \frac{h_{\text{slab}}}{4}$$

$$h_{\text{drop}} = 300 \text{ mm} ? \frac{200}{4}$$

$h_{\text{drop}} = 300 \text{ mm} > 50 \text{ mm}$  the projected thickness is ok.

- Check the length of drop panel

$$0.6 ? \frac{\ell}{6}$$

$$0.6 ? \frac{5}{6}$$

$$0.6 \text{ m} < 0.83 \text{ m not ok} \blacksquare$$

## 1.7 Minimum Thickness for two way slab

### 1.7.1 Minimum thickness for flat and plate slabs.

To ensure that slab deflections in service will not be troublesome, the best approach is to compute deflections with limiting value, methods have been developed that are both simple and acceptably accurate for predicting deflection of two-way slabs.

Alternatively deflection control can be achieved indirectly by adhering to more or less arbitrary limitation of minimum slab thickness, limitations developed from review of test data and study of the observed deflections of actual structures.

**ACI Code Table 8.3.1.1** establishes minimum thickness for two-way slabs; simplified criteria are included pertaining to slab thickness for flat and plate slabs (without interior beams).

**Table 8.3.1.1—Minimum thickness of nonpre-stressed two-way slabs without interior beams (mm)<sup>[1]</sup>**

$f_y$ , MPa <sup>[2]</sup>	Without drop panels <sup>[3]</sup>			With drop panels <sup>[3]</sup>		
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams <sup>[4]</sup>		Without edge beams	With edge beams <sup>[4]</sup>	
280	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$
420	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$
520	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$

<sup>[1]</sup> $\ell_n$  is the clear span in the long direction, measured face-to-face of supports (mm).

<sup>[2]</sup>For  $f_y$ , between the values given in the table, minimum thickness shall be calculated by linear interpolation.

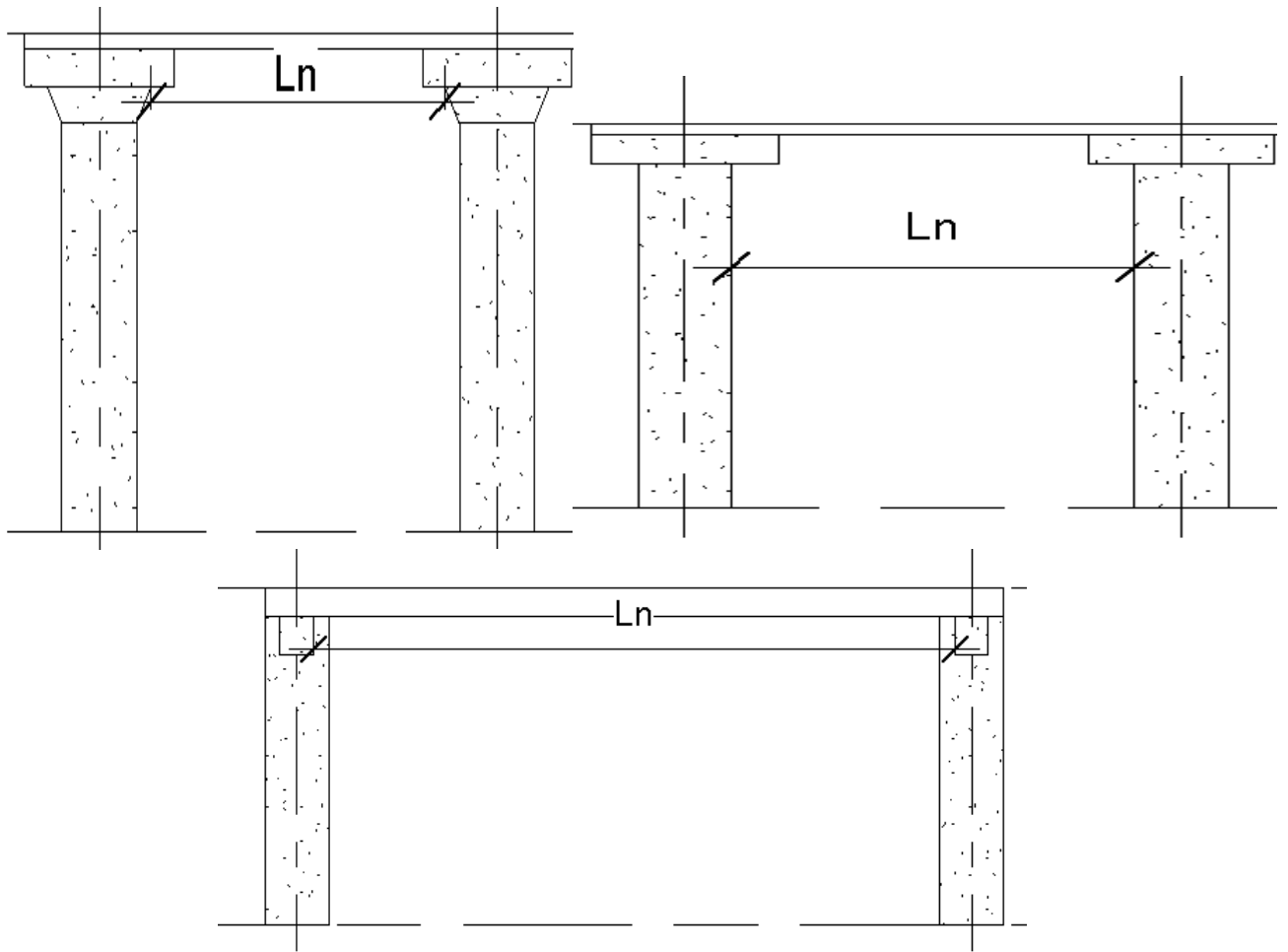
<sup>[3]</sup>Drop panels as given in 8.2.4.

<sup>[4]</sup>Slabs with beams between columns along exterior edges. Exterior panels shall be considered to be without edge beams if  $\alpha_f$  is less than 0.8. The value of  $\alpha_f$  for the edge beam shall be calculated in accordance with 8.10.2.7.

In all cases, the minimum thickness of slabs without interior beams **must not less than** the following:

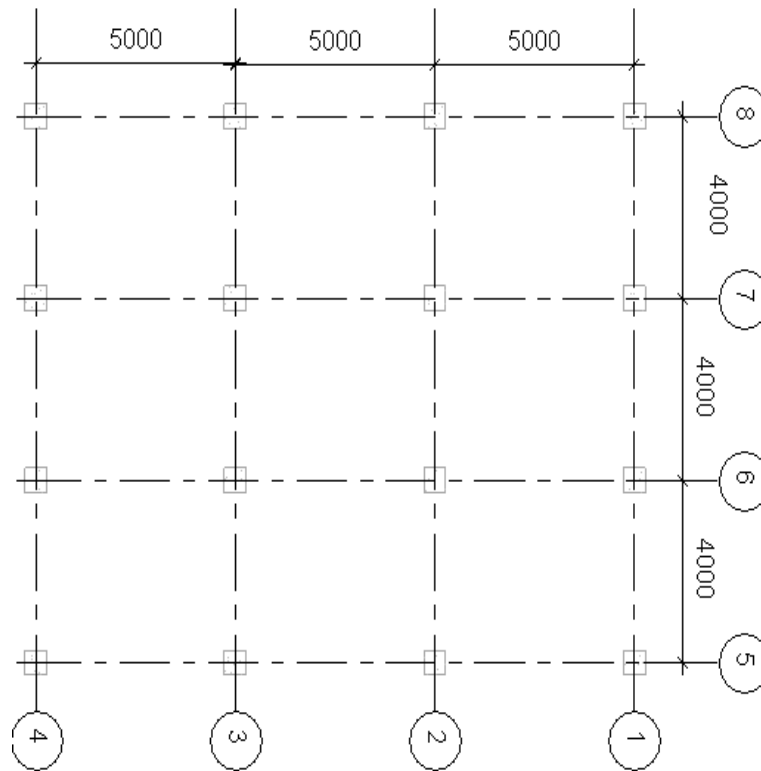
**For slabs without drop panels: 125mm**  
**For slabs with drop panels: 100 mm**

$\ell_n$  = is the length of the clear span in **the long direction** of two way slab, measured **face to face of the supports (column and capital column)** for slabs without beams and face of **beams or other supports (like walls)** in other cases. See the **Fig.1.11**.



**Fig 1.11:**  $l_n$  in two-way slab.

**Example 1:** Find the minimum required slab thickness according to ACI Code for building shown below in Fig. below, use  $f_y=420$  Mpa, the slab is without edge beams, Columns dimensions are (300x300) mm.



**Solution:**

**For exterior panel**

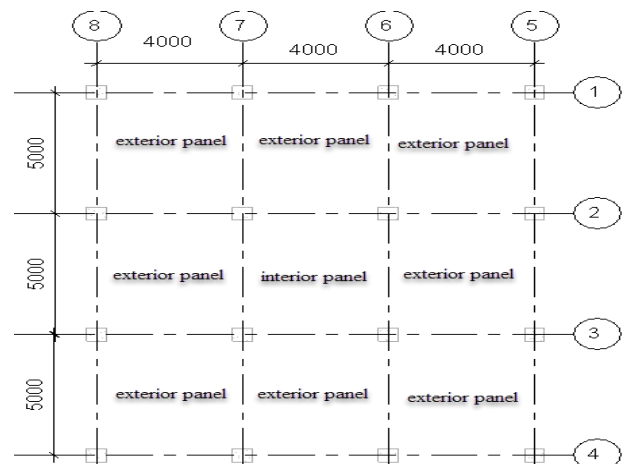
$$\ell_n = 5000 - 150 * 2$$

$$\ell_n = 4700 \text{ mm}$$

By using ACI code Table 8.3.1.1

Table 8.3.1.1—Minimum thickness of nonprestressed two-way slabs without interior beams (mm)<sup>[1]</sup>

$f_y$ , MPa <sup>[2]</sup>	Without drop panels <sup>[3]</sup>		With drop panels <sup>[3]</sup>			
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams <sup>[4]</sup>		Without edge beams	With edge beams <sup>[4]</sup>	
280	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$
420*	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$
520	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$



$$h_{\min} = \frac{\ell_n}{30} = \frac{4700}{30} = 156 \text{ mm} \approx 160 \text{ mm.}$$

**For interior panel**

$$h_{\min} = \frac{\ell_n}{33} = \frac{4700}{33} = 142 \text{ mm} \approx 150 \text{ mm.}$$

- In common practice we use same thickness for all slab panels
- Use the larger  $h=160 \text{ mm} > 125 \text{ mm}$  o.k

**Table 8.3.1.1—Minimum thickness of nonprestressed two-way slabs without interior beams (mm)<sup>[1]</sup>**

$f_y$ , MPa <sup>[2]</sup>	Without drop panels <sup>[3]</sup>			With drop panels <sup>[3]</sup>		
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams <sup>[4]</sup>		Without edge beams	With edge beams <sup>[4]</sup>	
280	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$
420	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$
520	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$

**Example 2:** resolve the previous example by assuming  $f_y=350 \text{ Mpa}$

**Solution:**

By using interpolation

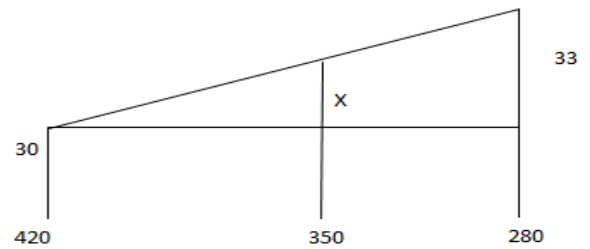
**For exterior panel**

$$\frac{33-30}{420-280} = \frac{X}{420-350}$$

$$X=1.5$$

$$\text{Total factor} = 30 + 1.5 = 31.5$$

$$h_{\min} = \frac{4700}{31.5} = 149 \text{ mm} \approx 150 \text{ mm}$$



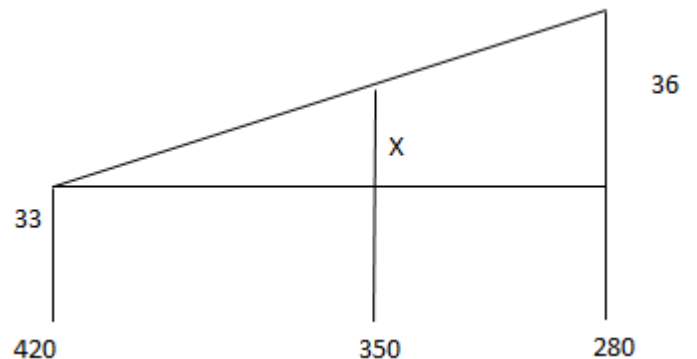
**For interior panel**

$$\frac{36-33}{420-280} = \frac{X}{420-350}$$

$$X=1.5$$

$$\text{Total factor} = 33 + 1.5 = 34.5$$

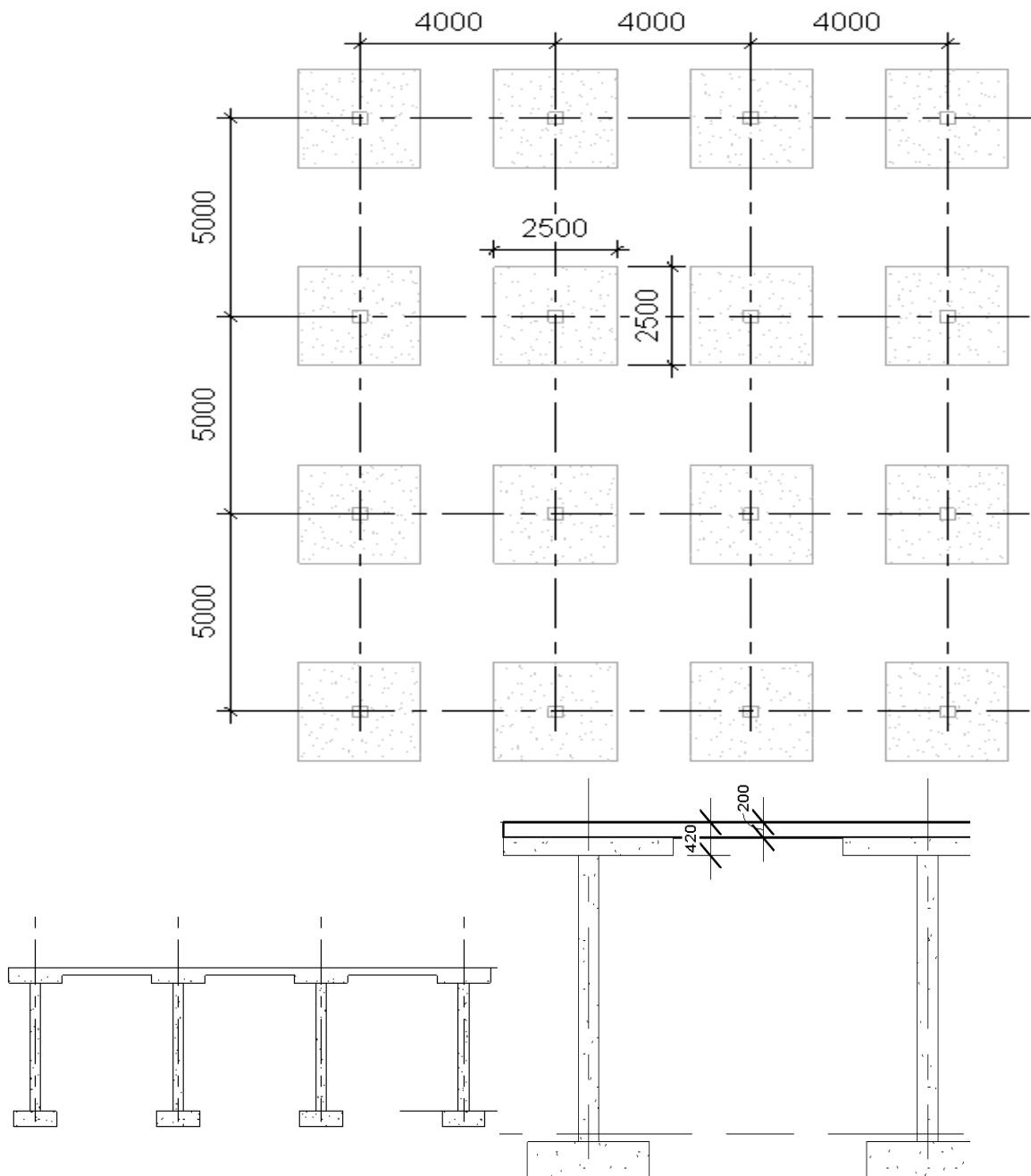
$$h_{\min} = \frac{4700}{34.5} = 136 \text{ mm} \approx 140 \text{ mm}$$



For all panels use the larger  $h=150 \text{ mm} > 125 \text{ mm}$  O.K



**Example 2:** Check the slab thickness and drop panel dimension according to ACI Code for building indicated in Fig. below, use  $f_y=420$  Mpa, slab thickness is 200 mm, drop panel thickness is 420 mm.



**Solution:**

Check drop panel thickness and dimensions

$$\frac{2500}{2} \geq \frac{\ell_1}{6} = \frac{4000}{6} = 666.66 \text{ mm}$$

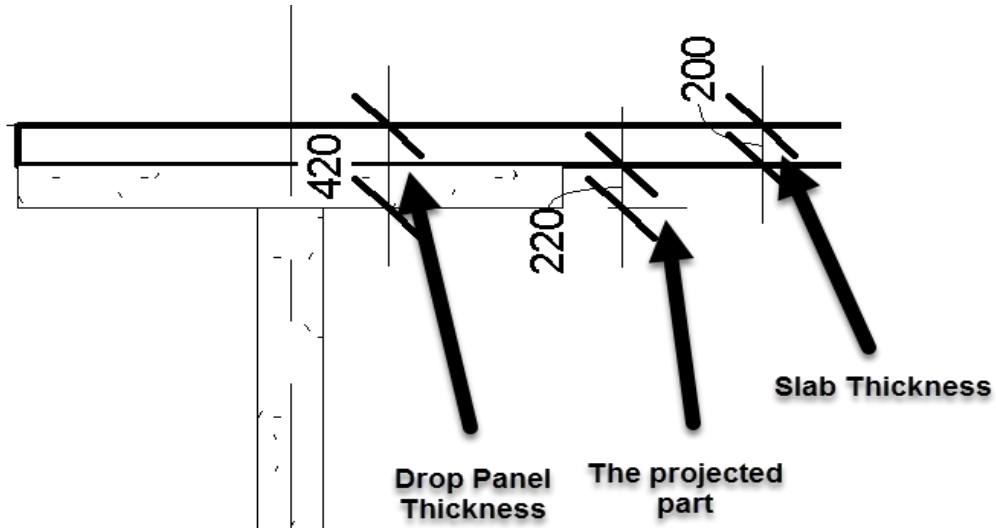
$$1250 \text{ mm} > 666.66 \text{ mm}$$

$$\frac{2500}{2} \geq \frac{\ell_2}{6} = \frac{5000}{6} = 833.33 \text{ mm}$$

$$1250 \text{ mm} > 833.33 \text{ mm}$$

The thickness is o.k.

-According to **ACI Code 8.2.4** the project part of drop panel shall be below the slab at least **one-fourth** of the slab thickness.



$$220 \geq \frac{h_{\text{slab}}}{4} = \frac{200}{4} = 50$$

220 mm > 50mm O.K.

### Check slab thickness

#### For exterior panel

$$\ell_n = 5000 - 150 * 2 = 4700 \text{ mm}$$

$$h_{\text{min}} = \frac{\ell_n}{33} = \frac{4700}{33} = 142 \text{ mm} \approx 145 \text{ mm}$$

200 > 145 the thickness is O.K.

**Table 8.3.1.1—Minimum thickness of nonprestressed two-way slabs without interior beams (mm)<sup>[1]</sup>**

$f_y$ , MPa <sup>[2]</sup>	Without drop panels <sup>[3]</sup>		With drop panels <sup>[3]</sup>			
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams <sup>[4]</sup>		Without edge beams	With edge beams <sup>[4]</sup>	
280	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$
420	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$
520	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$

**Table 8.3.1.1—Minimum thickness of nonprestressed two-way slabs without interior beams (mm)<sup>[1]</sup>**

$f_y$ , MPa <sup>[2]</sup>	Without drop panels <sup>[3]</sup>		With drop panels <sup>[3]</sup>			
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams <sup>[4]</sup>		Without edge beams	With edge beams <sup>[4]</sup>	
280	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$
420	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$
520	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$

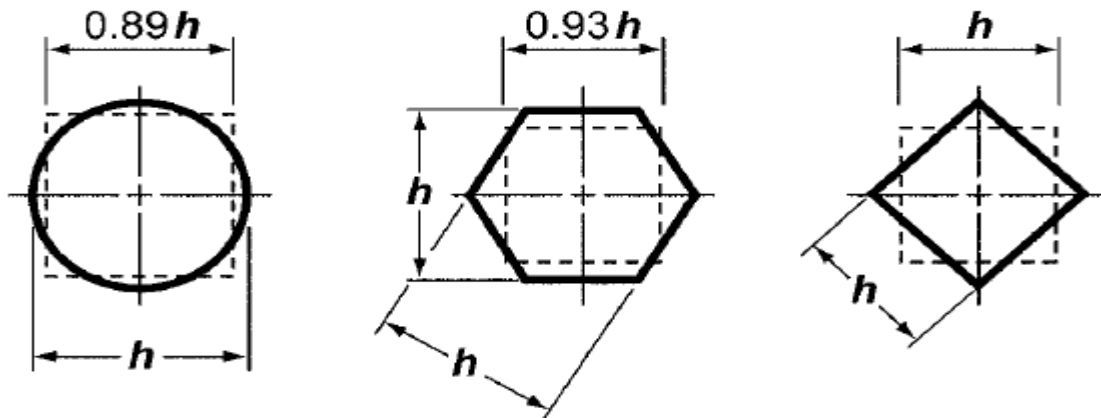
#### For interior panel

$$\ell_n = 5000 - 150 * 2 = 4700 \text{ mm}$$

$$h_{\text{min}} = \frac{\ell_n}{36} = \frac{4700}{36} = 130.5 \text{ mm} \approx 135 \text{ mm}$$

200 > 135 the thickness is O.K. ■

-According to **ACI Code 8.10.1.3** circular or polygon-shaped supports (**Columns and column capitals**) shall be treated as a **square supports** with same area.



**Example 4:** the architectural engineer assumes 200 mm slab thickness, check weather if this thickness is satisfying the deflection requirement of ACI Code, use  $f_y=420$  Mpa.

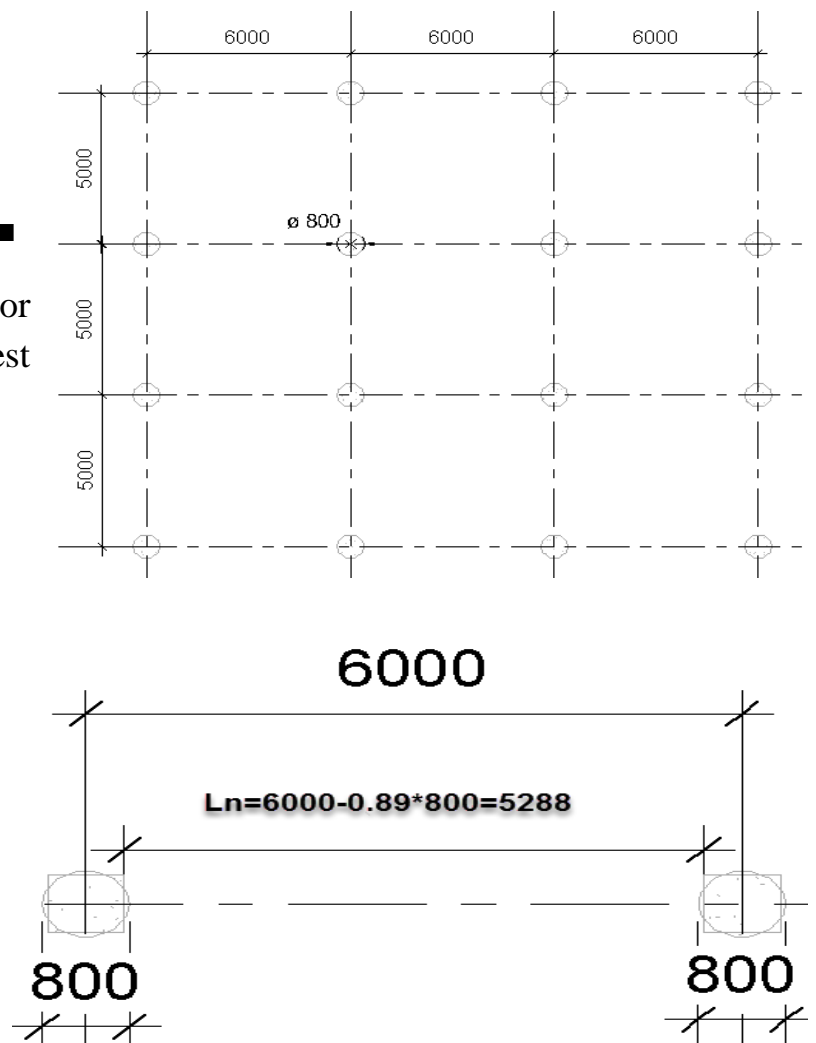
**Solution:**

$$\ell_n = 6000 - 0.89 \cdot 800 = 5288 \text{ mm}$$

$$h_{\min} = \frac{\ell_n}{30} = \frac{5288}{30} = 176.26 \text{ mm} \approx 180 \text{ mm}$$

200 mm > 180 mm the thickness is O.K. ■

**Note:** we check the thickness for exterior panel only, because it gives the largest thickness for deflection control.



**Example 5:** Check the slab thickness according to deflection requirement if the slab thickness is (180) mm, and the slab is supported by edge beam (300\*600) mm, columns (300 x 300) mm  $f_y=420$  Mpa.

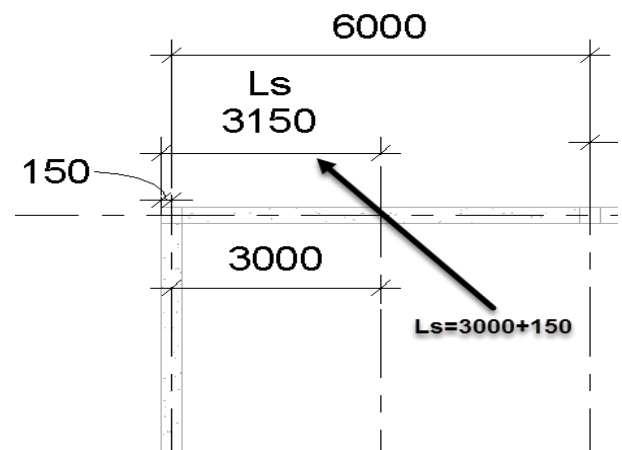
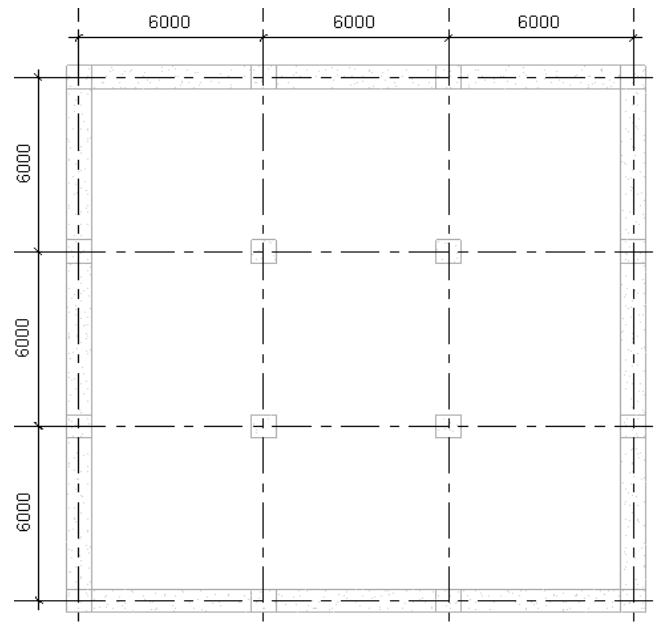
**Solution:**

- The slab is with edge beam.

$$\ell_n = 6000 - 2 \cdot 150 = 5700 \text{ mm}$$

$$h_{\min} = \frac{\ell_n}{33} = \frac{5700}{33} = 172.73 \text{ mm} \approx 175 \text{ mm}$$

180 mm > 175 mm the thickness is O.K ■



**Example 6:** Check the slab thickness according to deflection requirement for example 5 by assuming there are no edges beams if and slab thickness is (150) mm,.  $f_y=420$  Mpa.

**Solution:**

$$\ell_n = 6000 - 2 \cdot 150 = 5700 \text{ mm}$$

$$h_{\min} = \frac{\ell_n}{30} = \frac{5700}{30} = 190 \text{ mm}$$

150 mm < 190 mm the thickness is **not O.K.**

- Slab thickness is needed to be **increasing to 190** mm or larger. ■

**Example 6:** Find the thickness of the slab shown below, the slab is supported by edge beams and column capital diameter (1000) mm  $f_y=420$  Mpa

**Solution:**

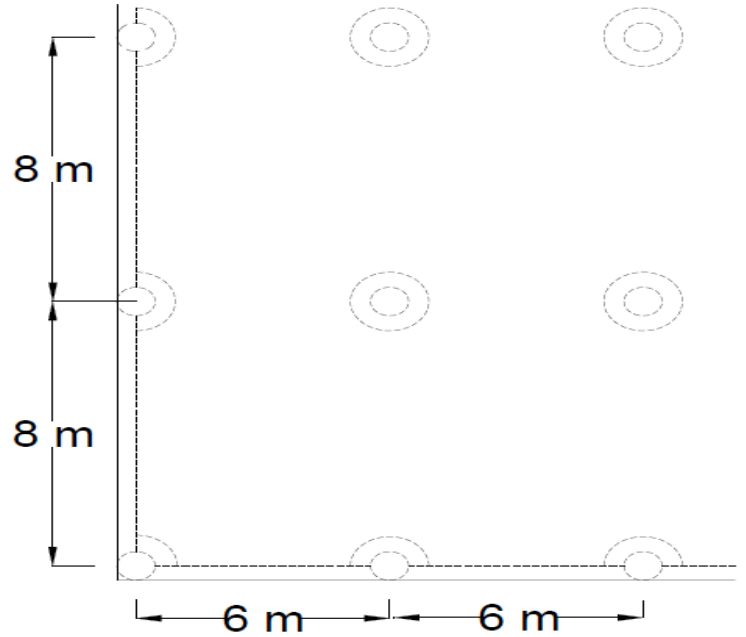
For exterior panel

- According to ACI Table 8.3.1.1

**Table 8.3.1.1—Minimum thickness of nonprestressed two-way slabs without interior beams (mm)<sup>[1]</sup>**

$f_y$ , MPa <sup>[2]</sup>	Without drop panels <sup>[3]</sup>		Interior panels	With drop panels <sup>[3]</sup>		Interior panels
	Exterior panels			Exterior panels	Interior panels	
	Without edge beams	With edge beams <sup>[4]</sup>	Without edge beams	With edge beams <sup>[4]</sup>	Interior panels	
280	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$	$\ell_n/40$
420	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$
520	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$	$\ell_n/34$

<sup>[1]</sup> $\ell_n$  is the clear span in the long direction, measured face-to-face of supports (mm).  
<sup>[2]</sup>For  $f_y$  between the values given in the table, minimum thickness shall be calculated by linear interpolation.  
<sup>[3]</sup>Drop panels as given in 8.2.4.



$$h = \frac{\ell_n}{33}$$

$$\ell_n = 8000 - 0.89 * 1000 = 7110 \text{ mm}$$

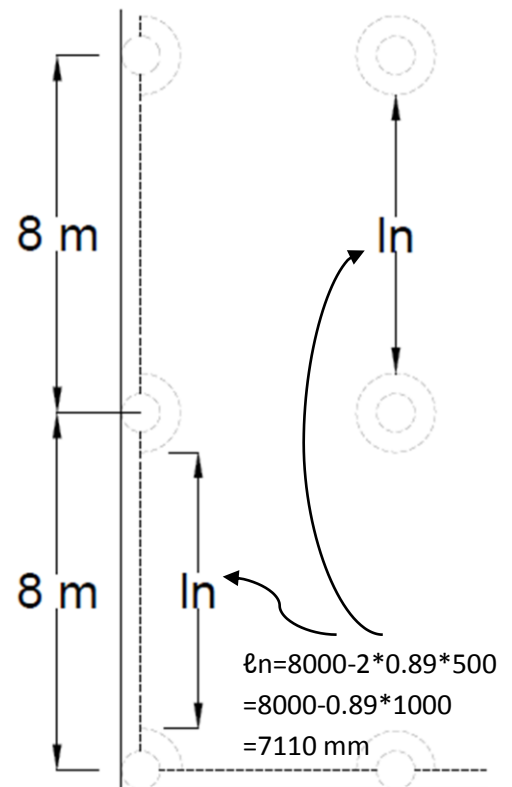
$$h = \frac{\ell_n}{33} = \frac{7110}{33} = 215.4 \text{ mm} \approx 220 \text{ mm} > 125 \text{ mm O.K}$$

For interior panel

**Table 8.3.1.1—Minimum thickness of nonprestressed two-way slabs without interior beams (mm)<sup>[1]</sup>**

$f_y$ , MPa <sup>[2]</sup>	Without drop panels <sup>[3]</sup>		Interior panels	With drop panels <sup>[3]</sup>		Interior panels
	Exterior panels			Exterior panels	Interior panels	
	Without edge beams	With edge beams <sup>[4]</sup>	Without edge beams	With edge beams <sup>[4]</sup>	Interior panels	
280	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$	$\ell_n/40$
420	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$
520	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$	$\ell_n/34$

<sup>[1]</sup> $\ell_n$  is the clear span in the long direction, measured face-to-face of supports (mm).  
<sup>[2]</sup>For  $f_y$  between the values given in the table, minimum thickness shall be calculated by linear interpolation.  
<sup>[3]</sup>Drop panels as given in 8.2.4.  
<sup>[4]</sup>Slabs with beams between columns along exterior edges. Exterior panels shall be considered to be without edge beams if  $\alpha_f$  is less than 0.8. The value of  $\alpha_f$  for the edge beam shall be calculated in accordance with 8.10.2.7.



$$h = \frac{\ell_n}{33}$$

$$\ell_n = 8000 - 0.89 * 1000 = 7110 \text{ mm}$$

$$h = \frac{\ell_n}{33} = \frac{7110}{33} = 215.4 \text{ mm} \approx 220 \text{ mm} > 125 \text{ mm O.K}$$

Use  $h=220$  mm ■

**Example 6:** Find the thickness of the slab shown below, the slab is without edge beams and column capital diameter (1000) mm  $f_y=420$  Mpa

**Solution:**

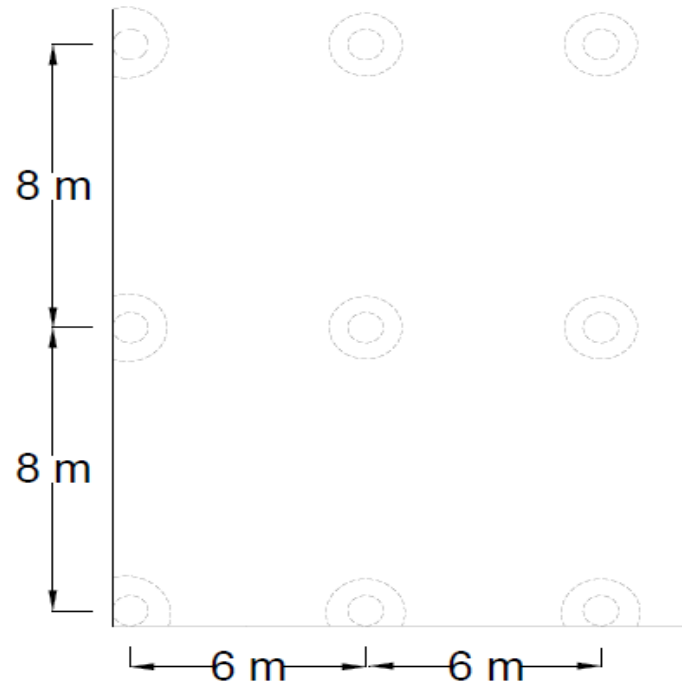
**For exterior panel**

- According to ACI Table 8.3.1.1

**Table 8.3.1.1—Minimum thickness of nonprestressed two-way slabs without interior beams (mm)<sup>[1]</sup>**

$f_y$ , MPa <sup>[2]</sup>	Without drop panels <sup>[3]</sup>			With drop panels <sup>[3]</sup>		
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams <sup>[4]</sup>		Without edge beams	With edge beams <sup>[4]</sup>	
280	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$
420	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$
520	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$

<sup>[1]</sup> $\ell_n$  is the clear span in the long direction, measured face-to-face of supports (mm).  
<sup>[2]</sup>For  $f_y$  between the values given in the table, minimum thickness shall be calculated by linear interpolation.  
<sup>[3]</sup>Drop panels as given in 8.2.4.  
<sup>[4]</sup>Slabs with beams between columns along exterior edges. Exterior panels shall be considered to be without edge beams if  $\alpha_f$  is less than 0.8. The value of  $\alpha_f$  for the edge beam shall be calculated in accordance with 8.10.2.7.



$$h = \frac{\ell_n}{30}$$

$$\ell_n = 8000 - 0.89 * 1000 = 7110 \text{ mm}$$

$$h = \frac{\ell_n}{30} = \frac{7110}{30} = 237 > 125 \text{ mm}$$

**For interior panel**

**Table 8.3.1.1—Minimum thickness of nonprestressed two-way slabs without interior beams (mm)<sup>[1]</sup>**

$f_y$ , MPa <sup>[2]</sup>	Without drop panels <sup>[3]</sup>			With drop panels <sup>[3]</sup>		
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams <sup>[4]</sup>		Without edge beams	With edge beams <sup>[4]</sup>	
280	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$
420	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$
520	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$

<sup>[1]</sup> $\ell_n$  is the clear span in the long direction, measured face-to-face of supports (mm).  
<sup>[2]</sup>For  $f_y$  between the values given in the table, minimum thickness shall be calculated by linear interpolation.  
<sup>[3]</sup>Drop panels as given in 8.2.4.  
<sup>[4]</sup>Slabs with beams between columns along exterior edges. Exterior panels shall be considered to be without edge beams if  $\alpha_f$  is less than 0.8. The value of  $\alpha_f$  for the edge beam shall be calculated in accordance with 8.10.2.7.

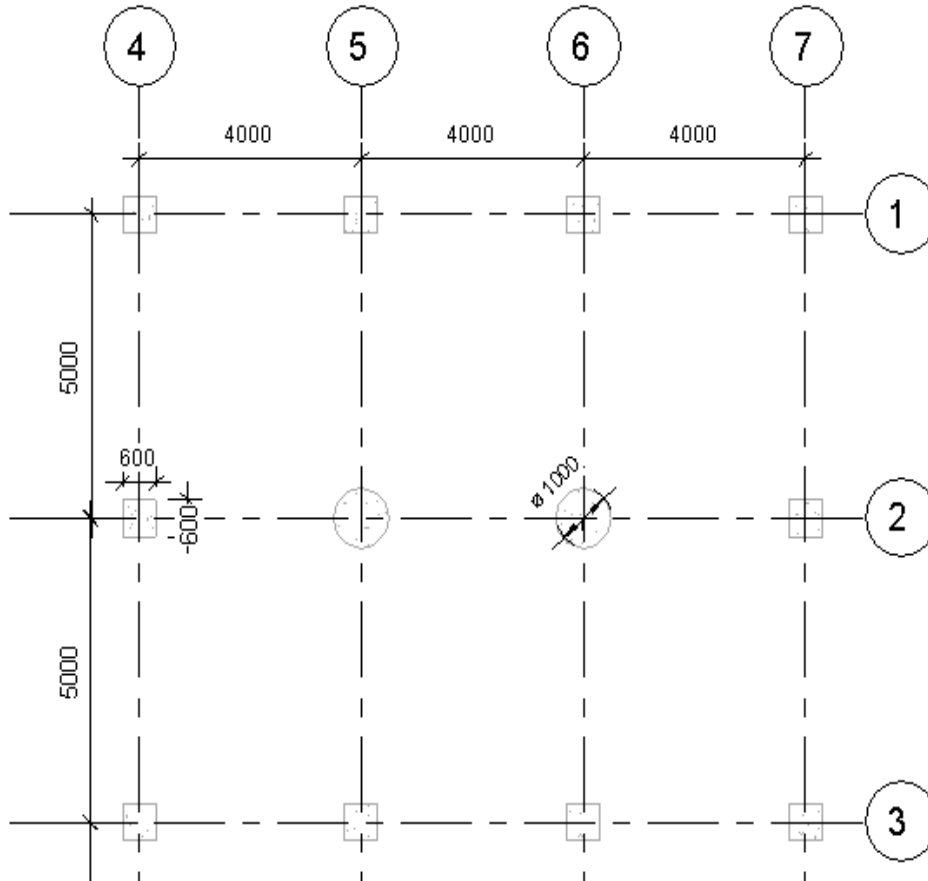
$$h = \frac{\ell_n}{33}$$

$$\ell_n = 8000 - 0.89 * 1000 = 7110 \text{ mm}$$

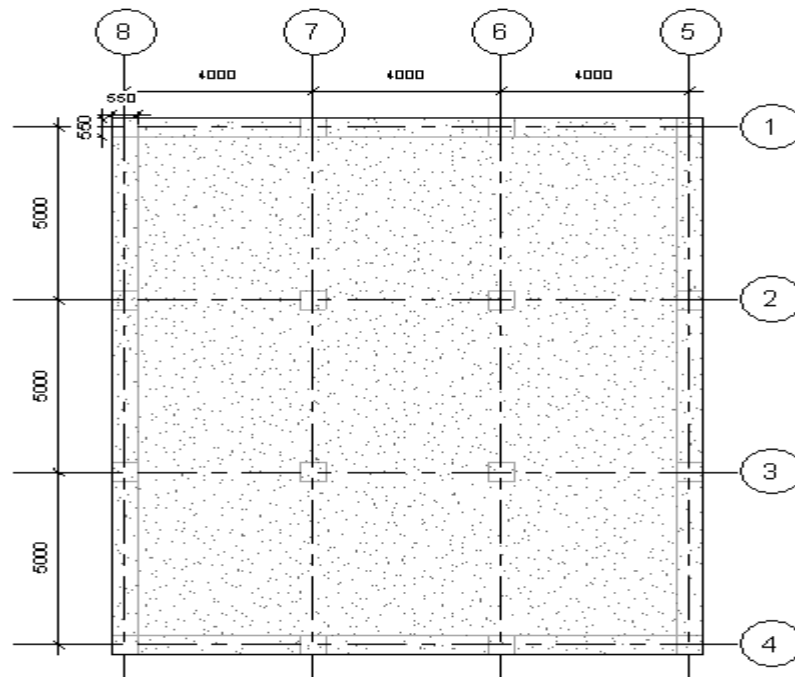
$$h = \frac{\ell_n}{33} = \frac{7110}{33} = 215.4 \text{ mm} > 125 \text{ mm}$$

Use  $h = 240 \text{ mm}$  ■

**H.W 1:** Find the minimum required slab thickness according to ACI Code for the slab shown below Fig. use  $f_y=420$  Mpa.



**H.W 2:** Find the minimum required slab thickness according to ACI Code for the slab shown below Fig. use  $f_y=420$  Mpa. Beam Dimensions are (300x600) mm.



## 1.5.2 Minimum Thickness for Two-Way Slab with Interior Beams.

The parameter used to define the relative stiffness of the beam and slab spanning in either direction is  $\alpha_f$ , it can be calculated by equation below:

$$\alpha_f = \frac{E_{cb} * I_b}{E_{cs} * I_s}$$

In which  $E_{cb}$  and  $E_{cs}$  are the moduli of elasticity of the beam and slab concrete (**usually the same**) and  $I_b$  and  $I_s$  are the moments of inertia of the effective beam and the slab.

The moment of inertia of a flange beam about its own centroid axis can be computed based on simple definition of centroid or by approximate method.

- According to ACI code, the two-way slab with interior beams can be found by Table 8.3.1.2

**Table 8.3.1.2—Minimum thickness of nonpre-stressed two-way slabs with beams spanning between supports on all sides**

$\alpha_{fm}$ <sup>[1]</sup>	Minimum $h$ , mm		
$\alpha_{fm} \leq 0.2$	8.3.1.1 applies		(a)
$0.2 < \alpha_{fm} \leq 2.0$	Greater of:	$\frac{\ell_n \left( 0.8 + \frac{f_y}{1400} \right)}{36 + 5\beta (\alpha_{fm} - 0.2)}$	(b) <sup>[2],[3]</sup>
		125	(c)
$\alpha_{fm} > 2.0$	Greater of:	$\frac{\ell_n \left( 0.8 + \frac{f_y}{1400} \right)}{36 + 9\beta}$	(d) <sup>[2],[3]</sup>
		90	(e)

<sup>[1]</sup> $\alpha_{fm}$  is the average value of  $\alpha_f$  for all beams on edges of a panel and  $\alpha_f$  shall be calculated in accordance with 8.10.2.7.

<sup>[2]</sup> $\ell_n$  is the clear span in the long direction, measured face-to-face of beams (mm).

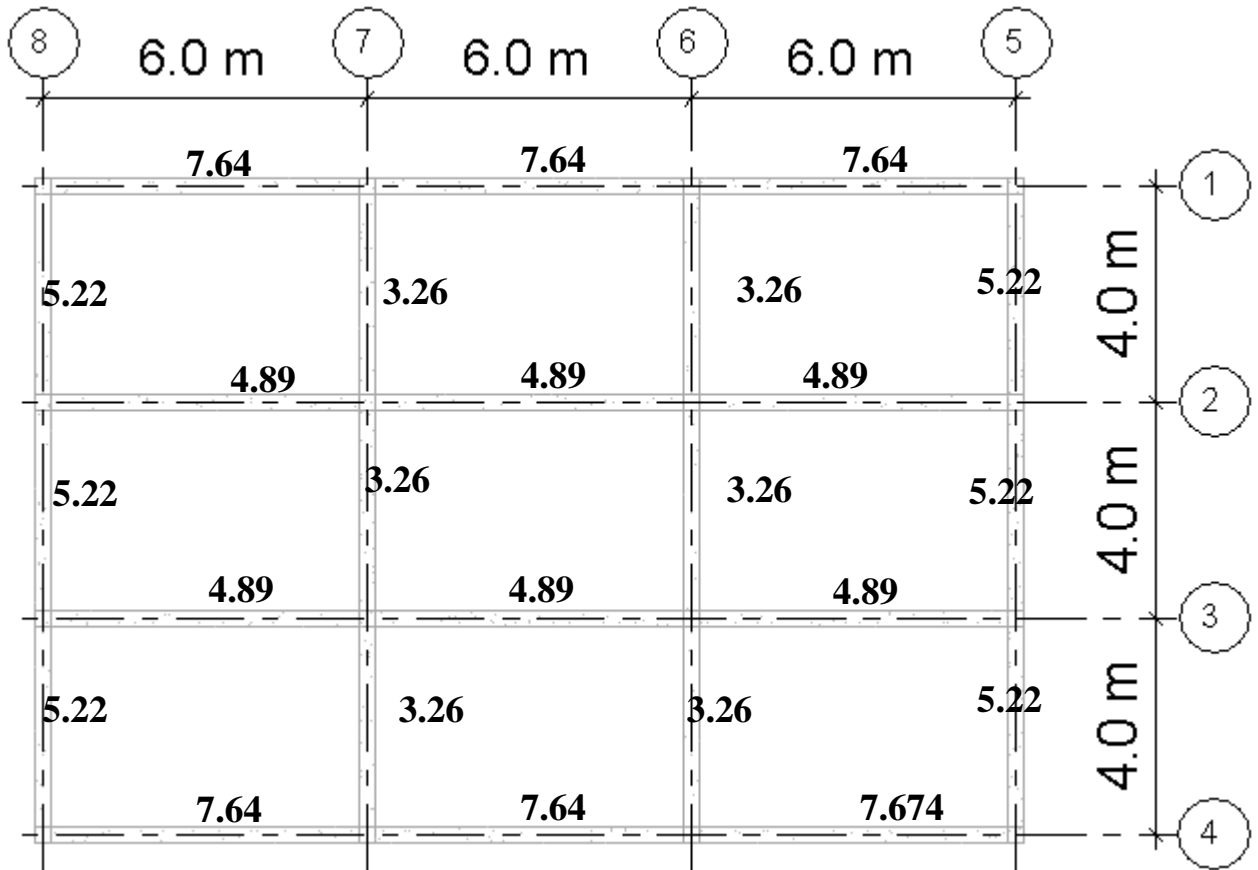
<sup>[3]</sup> $\beta$  is the ratio of clear spans in long to short directions of slab.

### Notes:

- $\beta = \frac{\text{clear span in the long direction } \ell_n}{\text{clear span in the short direction } s_n}$
- Both of  $\ell_n$  and  $s_n$  are measured from **face of beams**.
- $\alpha_{fm}$  is the **average value** of  $\alpha_f$  for all beams on edges of a panel.



**Example:** For the slab with beams shown below in Figure is satisfactory, use  $f_y=420$  Mpa, beam dimension (300x600) mm,  $\alpha_{fl}$  for each beams are indicated in figure below.



**Solution:**

Find  $\alpha_{fm}$  for each panel:

**Panel 1**

$$\alpha_{fm} = \frac{4.89 + 4.89 + 3.26 + 3.26}{4} = 4.075$$

$$\alpha_{fm} > 2$$

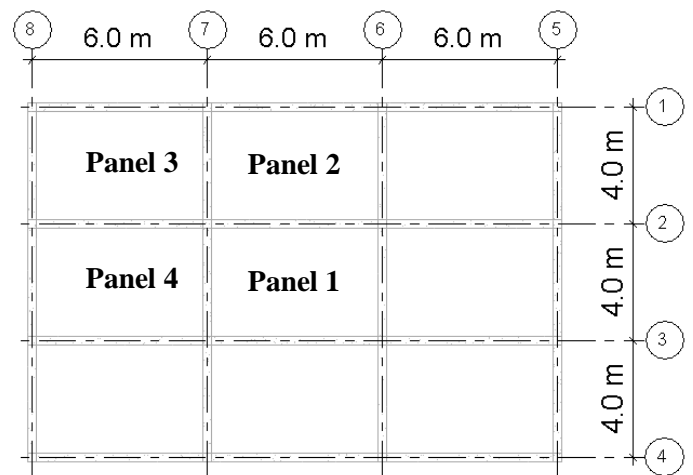
- By using the equation in ACI Code Table 8.3.1.2:

$$h = \frac{\ell n \left(0.8 + \frac{f_y}{1400}\right)}{36 + 9\beta} \text{ or } 90 \text{ (choose larger)}$$

$$\beta = \frac{\ell n}{sn} = \frac{6000 - 300}{4000 - 300} = \frac{5700}{3700} = 1.54$$

$$h = \frac{\ell n \left(0.8 + \frac{f_y}{1400}\right)}{36 + 9\beta} = \frac{5700 \left(0.8 + \frac{420}{1400}\right)}{36 + 9 \times 1.54} = 125.75 \text{ mm} > 90 \text{ mm O.K}$$

Use  $h \approx 130$  mm



$\alpha_{fm}^{(1)}$	Minimum $h$ , mm		
$\alpha_{fm} \leq 0.2$	8.3.1.1 applies		(a)
$0.2 < \alpha_{fm} \leq 2.0$	Greater of:	$\frac{\ell_n \left(0.8 + \frac{f_y}{1400}\right)}{36 + 5\beta(\alpha_{fm} - 0.2)}$	(b) <sup>(2),(3)</sup>
		125	(c)
$\alpha_{fm} > 2.0$	Greater of:	$\frac{\ell_n \left(0.8 + \frac{f_y}{1400}\right)}{36 + 9\beta}$	(d) <sup>(2),(3)</sup>
		90	(e)

### **Panel 2**

$$\alpha_{fm} = \frac{7.64 + 3.26 + 3.26 + 4.89}{4} = 4.76$$

$$\alpha_{fm} > 2$$

$$h = \frac{\ln(0.8 + \frac{fy}{1400})}{36 + 9\beta} \text{ or } 90$$

$$\beta = \frac{\ln}{sn} = \frac{6000 - 300}{4000 - 300} = \frac{5700}{3700} = 1.54$$

$$h = \frac{\ln(0.8 + \frac{fy}{1400})}{36 + 9\beta} = \frac{5700(0.8 + \frac{420}{1400})}{36 + 9 \cdot 1.54} = 125.75 \text{ mm} > 90 \text{ mm O.K}$$

Use  $h \approx 130 \text{ mm}$

### **Panel 3**

$$\alpha_{fm} = \frac{7.64 + 3.26 + 5.22 + 4.89}{4} = 5.25$$

$$\alpha_{fm} > 2$$

$$h = \frac{\ln(0.8 + \frac{fy}{1400})}{36 + 9\beta} \text{ or } 90$$

$$\beta = \frac{\ln}{sn} = \frac{6000 - 300}{4000 - 300} = \frac{5700}{3700} = 1.54$$

$$h = \frac{\ln(0.8 + \frac{fy}{1400})}{36 + 9\beta} = \frac{5700(0.8 + \frac{420}{1400})}{36 + 9 \cdot 1.54} = 125.75 \text{ mm} > 90 \text{ mm O.K}$$

Use  $h \approx 130 \text{ mm}$

### **Panel 4**

$$\alpha_{fm} = \frac{4.89 + 3.26 + 5.22 + 4.89}{4} = 4.565$$

$$\alpha_{fm} > 2$$

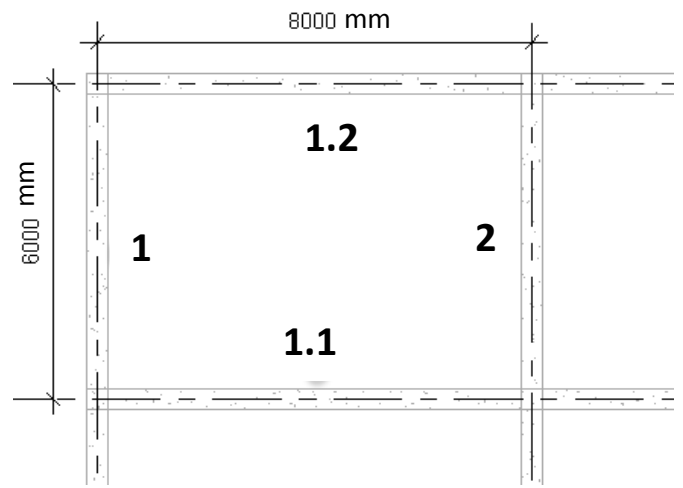
$$h = \frac{\ln(0.8 + \frac{fy}{1400})}{36 + 9\beta} \text{ or } 90$$

$$\beta = \frac{\ln}{sn} = \frac{6000 - 300}{4000 - 300} = \frac{5700}{3700} = 1.54$$

$$h = \frac{\ln(0.8 + \frac{fy}{1400})}{36 + 9\beta} = \frac{5700(0.8 + \frac{420}{1400})}{36 + 9 \cdot 1.54} = 125.75 \text{ mm} > 90 \text{ mm O.K}$$

Use  $h \approx 130 \text{ mm}$  for all slab panels ■

**Example:** Find slab thickness for the following panel,  $\alpha_f$  for each beams are indicated in Fig. below,  $F_y=420$  Mpa. Beam dimensions are (400x600) mm.



**Solution:**

Find  $\alpha_{fm}$  for the panel:

**Panel 1**

$$\alpha_{fm} = \frac{1+1.1+2+1.2}{4} = 1.325$$

$$0.2 < \alpha_{fm} \leq 2$$

- By using the equation in ACI Code Table 8.3.1.2:

$$h = \frac{\ell_n \left( 0.8 + \frac{f_y}{1400} \right)}{36 + 5\beta(\alpha_{fm} - 0.2)} \text{ or } 125$$

$$\beta = \frac{\ell_n}{s_n} = \frac{8000 - 400}{6000 - 400} = \frac{7600}{5600} = 1.357$$

$$h = \frac{\ell_n \left( 0.8 + \frac{f_y}{1400} \right)}{36 + 5\beta(\alpha_{fm} - 0.2)} = \frac{7600 \left( 0.8 + \frac{420}{1400} \right)}{36 + 5 \cdot 1.357(1.325 - 0.2)} = 191.6 \text{ mm} > 125 \text{ mm O.K}$$

Use  $h \approx 200$  mm ■

$\alpha_{fm}^{[1]}$	Minimum $h$ , mm		
$\alpha_{fm} \leq 0.2$	8.3.1.1 applies		(a)
$0.2 < \alpha_{fm} \leq 2.0$	Greater of:	$\ell_n \left( \frac{0.8 + \frac{f_y}{1400}}{36 + 5\beta(\alpha_{fm} - 0.2)} \right)$	(b) <sup>[2],[3]</sup>
		125	(c)
$\alpha_{fm} > 2.0$	Greater of:	$\ell_n \left( \frac{0.8 + \frac{f_y}{1400}}{36 + 9\beta} \right)$	(d) <sup>[2],[3]</sup>
		90	(e)

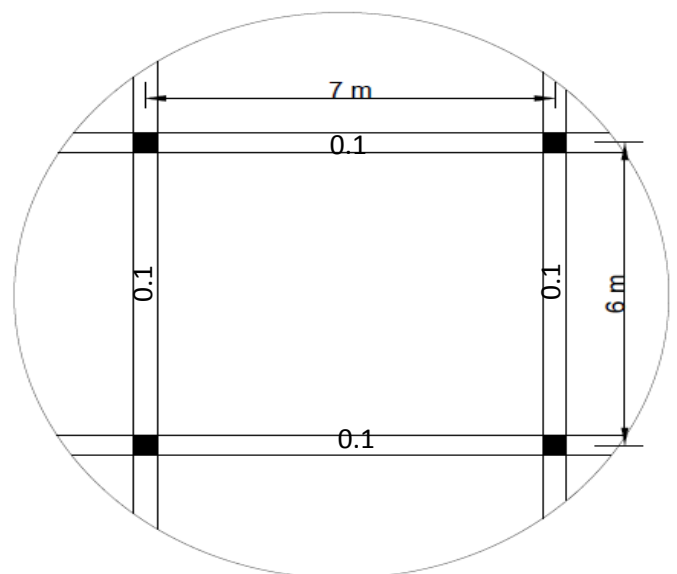
**Example:** Find slab thickness for the interior panel that showing if figure below,  $\alpha_f$  for all beams are 0.1,  $f_y=420$  Mpa. Beams and columns dimensions are (300x300) mm.

**Solution:**

Find  $\alpha_{fm}$

$$\alpha_{fm} = \frac{0.1+0.1+0.1+0.1}{4} = 0.1 < 2$$

$\alpha_{fm}^{[1]}$	Minimum $h$ , mm		
$\alpha_{fm} \leq 0.2$	8.3.1.1 applies		(a)
$0.2 < \alpha_{fm} \leq 2.0$	Greater of:	$\ell_n \left( \frac{0.8 + \frac{f_y}{1400}}{36 + 5\beta(\alpha_{fm} - 0.2)} \right)$	(b) <sup>[2],[3]</sup>
		125	(c)
$\alpha_{fm} > 2.0$	Greater of:	$\ell_n \left( \frac{0.8 + \frac{f_y}{1400}}{36 + 9\beta} \right)$	(d) <sup>[2],[3]</sup>
		90	(e)



- Since  $\alpha_{fm}$  for the panel is **less than or equal to 0.2**, then the slab is considered without beams and **ACI Table 8.3.1.1** must apply

**Table 8.3.1.1—Minimum thickness of nonprestressed two-way slabs without interior beams (mm)<sup>[1]</sup>**

$f_y$ , MPa <sup>[2]</sup>	Without drop panels <sup>[3]</sup>			With drop panels <sup>[3]</sup>		
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams <sup>[4]</sup>		Without edge beams	With edge beams <sup>[4]</sup>	
280	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$
420	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$
520	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$

$$h = \frac{\ell_n}{33}$$

$$\ell_n = 7000 - 300 = 6700 \text{ mm}$$

$$h = \frac{6700}{33} = 203 \text{ mm} \approx 210 \text{ mm} > 125 \text{ mm} \quad \blacksquare$$

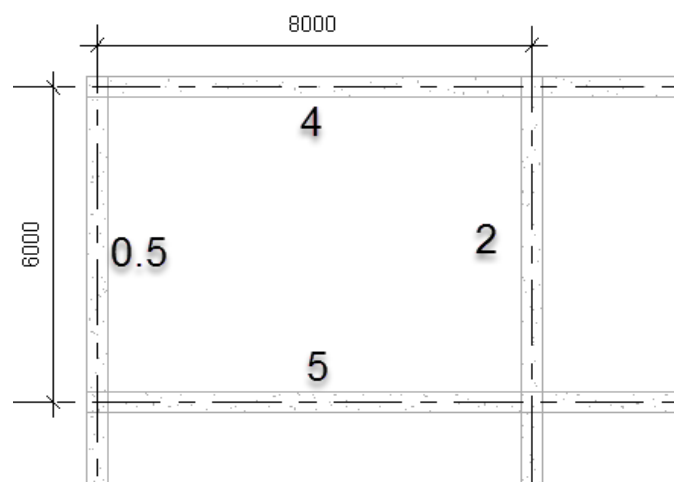
**Note:** According to **ACI Code 8.3.1.2.1** at discontinuous edges of slabs, if an edge beam with  $\alpha_f < 0.8$  then the minimum thickness required by (b) or (d) of **Table 8.3.1.2** shall be increased at least **10 percent** in the panel with discontinuous edge.

**Example:** Find slab thickness for the following panel,  $\alpha_f$  for each beam are indicated in Fig. below,  $F_y = 420$ . Beam dimensions are (400x600) mm.

**Solution:**

$$\alpha_{fm} = \frac{4 + 2 + 5 + 0.5}{4} = 2.875$$

$$\alpha_{fm} = 2.875 > 2$$



By using the equation in **ACI Code Table (8.3.1.2)**:

$$h = \frac{\ell_n \left( 0.8 + \frac{f_y}{1400} \right)}{36 + 9\beta} \text{ Or } 90 \text{ (greater)}$$

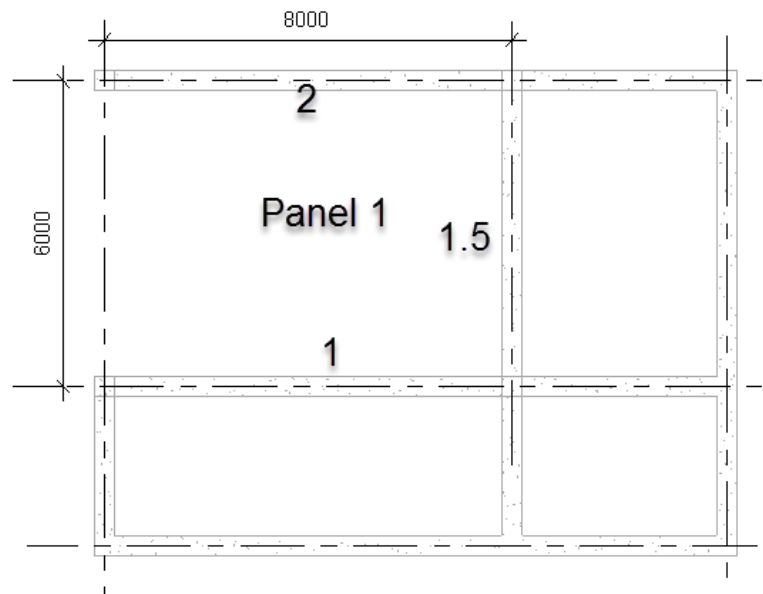
$\alpha_m^{[1]}$	Minimum $h$ , mm		
$\alpha_m \leq 0.2$	8.3.1.1 applies		(a)
$0.2 < \alpha_m \leq 2.0$	Greater of:	$\ell_n \left( \frac{0.8 + \frac{f_y}{1400}}{36 + 5\beta(\alpha_m - 0.2)} \right)$	(b) <sup>[2],[3]</sup>
		125	(c)
$\alpha_m > 2.0$	Greater of:	$\ell_n \left( \frac{0.8 + \frac{f_y}{1400}}{36 + 9\beta} \right)$	(d) <sup>[2],[3]</sup>
		90	(e)

- Since  $\alpha_f$  for edge beam **0.5 < 0.8**, then the equation shall be increased by **10 percent**.

$$h = 1.1 * \frac{\ell_n \left( 0.8 + \frac{f_y}{1400} \right)}{36 + 9\beta}, \beta = \frac{\text{clear span in long direction}}{\text{clear span in short direction}} = \frac{8000 - 400}{6000 - 400} = \frac{7600}{5600} = 1.36$$

$$h = 1.1 * \frac{7600 \left( 0.8 + \frac{420}{1400} \right)}{36 + 9 * 1.36} = 190.63 > 90 \text{ use } h \approx 200 \text{ mm} \blacksquare$$

**H.W:** Find slab thickness for the panel 1,  $\alpha_f$  for each beams are indicated in Fig. below,  $F_y=420$ . Beam digestions are (300x600) mm.



# General Examples for calculating slab thickness

**Example 1:** Find the minimum thickness of a slab for an interior panels due to deflection control for the following: Use  $f_y=350$  Mpa

- Slab with beams (8.1 x 8.2) m clear span with  $\alpha_{fm}=2.3$
- Flat plate (4.4 x 4.6) m clear span.
- Flat slab with drop panels (6.2 x 6.2) m clear span.

## Solution:

**Slab with beams (8.1 x 8.2) m clear span with  $\alpha_{fm}=2.3$**

$$\alpha_{fm}=2.3 > 2$$

$$h = \frac{\ell_n \left(0.8 + \frac{f_y}{1400}\right)}{36 + 9\beta}$$

$$\beta = \frac{\ell_n}{s_n} = \frac{8200}{8100} = 1.012$$

$$h = \frac{8200 \cdot \left(0.8 + \frac{350}{1400}\right)}{36 + 9 \cdot 1.012} = 190.87 \text{ mm} > 90 \text{ mm O.K}$$

**Flat plate (4.4 x 4.6) m clear span.**

From ACI Table 8.3.1.1

$f_y$ , MPa <sup>[2]</sup>	Without drop panels <sup>[3]</sup>			With drop panels <sup>[3]</sup>		
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams <sup>[4]</sup>		Without edge beams	With edge beams <sup>[4]</sup>	
280	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$
420	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$
520	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$

For  $f_y=280$  Mpa  $\frac{\ell_n}{36}$

For  $f_y=420$  Mpa  $\frac{\ell_n}{33}$

For  $f_y=350$  Mpa  $\frac{\ell_n}{34.5}$  (by linear interpolation)

$$h = \frac{\ell_n}{34.5} = \frac{4600}{34.5} = 133.33 \text{ mm} > 125 \text{ mm O.k}$$

**c. Flat slab with drop panels (6.2 x 6.2) m clear span.**

From ACI Table 8.3.1.1

$f_y$ , MPa <sup>[2]</sup>	Without drop panels <sup>[3]</sup>			With drop panels <sup>[3]</sup>		
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams <sup>[4]</sup>		Without edge beams	With edge beams <sup>[4]</sup>	
280	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$
420	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$
520	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$

350 →

For  $f_y=280$  Mpa  $\frac{\ell_n}{40}$

For  $f_y=420$  Mpa  $\frac{\ell_n}{36}$

For  $f_y=350$  Mpa  $\frac{\ell_n}{38}$  (by linear interpolation)

$h = \frac{\ell_n}{38} = \frac{6200}{38} = 163.15 \text{ mm} > 100 \text{ mm O.k.} \blacksquare$

**Example 2:** Find the minimum thickness of a slab for an interior panels due to deflection control for the following: Use  $f_y=420$  Mpa

- Slab with beams ( 8.2 x 7.7) m clear span with  $\alpha_{fm}=2.3$
- Slab without drop panels ( 5.4 x 4.8) m clear span with  $\alpha_{fm}=0.18$
- Flat plate (4.2 x 4.6) m clear span.
- Flat slab with drop panels (6 x 6.2) m clear span.
- Slab with beams (5.8 x 5.8) m clear span with  $\alpha_{fm}=1.5$

**Solution:**

**a. Slab with beams ( 8.2 x 7.7) m clear span with  $\alpha_{fm}=2.3$**

$\alpha_{fm}=2.3 > 2$

$$h = \frac{\ell_n(0.8 + \frac{f_y}{1400})}{36 + 9\beta}$$

$\beta = \frac{\ell_n}{s_n} = \frac{8200}{7700} = 1.065$

$h = \frac{8200*(0.8 + \frac{420}{1400})}{36 + 9*1.065} = 197.87 \text{ mm} > 90 \text{ mm O.k}$

Use  $h \approx 200 \text{ mm}$

**b. Slab without drop panels ( 5.4 x 4.8) m clear span with  $\alpha_{fm}=0.18$**

$$\alpha_{fm}=0.18 < 0.2$$

Then the slab is considered without beams, use **ACI Table 8.3.1.1**

$$h = \frac{\ell n}{33} = \frac{5400}{33} = 163.65 \text{ mm} > 125 \text{ mm O.K}$$

Use  $h \approx 170 \text{ mm}$

**c. Flat plate (4.2 x 4.6) m clear span.**

From **ACI Table 8.3.1.1**

$$h = \frac{\ell n}{33} = \frac{4600}{33} = 139.4 \text{ mm} > 125 \text{ mm O.K}$$

Use  $h \approx 140 \text{ mm}$

**d. Flat slab with drop panels (6 x 6.2) m clear span.**

From **ACI Table 8.3.1.1**

$$h = \frac{\ell n}{36} = \frac{6200}{36} = 172.2 \text{ mm} > 100 \text{ mm O.K}$$

Use  $h \approx 175 \text{ mm}$

**e. Slab with beams (5.8 x 5.8) m clear span with  $\alpha_{fm}=1.5$**

$$0.2 < \alpha_{fm}=1.5 < 2$$

$$h = \frac{\ell n \left( 0.8 + \frac{f_y}{1400} \right)}{36 + 5\beta(\alpha_{fm} - 0.2)}$$

$$\beta = \frac{\ell n}{s n} = \frac{5800}{5800} = 1$$

$$h = \frac{5800 * \left( 0.8 + \frac{420}{1400} \right)}{36 + 5 * 1 * (1.5 - 0.2)} = 150.12 \text{ mm} > 125 \text{ mm O.K}$$

Use  $h \approx 160 \text{ mm}$  ■



- Example 3:** Find the minimum thickness of a slab for an interior panels due to deflection control for the following: Use  $f_y=420$  Mpa (60000 psi)
- Flat slab with drop panels (6.4 x 6.4) m clear span
  - Slab with beams (6.0 x 6.0) m clear span with  $\alpha_{fm}=2.7$
  - Slab with beams (5.4 x 4.5) m clear span with  $\alpha_{fm}=0.7$ .
  - Flat plate (4.6 x 4.6) m clear span.

**Solution:**

**a. Flat slab with drop panels (6.4 x 6.4) m clear span**

From ACI Table 8.3.1.1

$$h = \frac{\ell_n}{36} = \frac{6400}{36} = 177.78 \text{ mm} > 100 \text{ mm O.K}$$

Use  $h \approx 180$  mm

**b. Slab with beams (6.0 x 6.0) m clear span with  $\alpha_{fm}=2.7$**

$$h = \frac{\ell_n \left( 0.8 + \frac{f_y}{1400} \right)}{36 + 9\beta}$$

$$\beta = \frac{\ell_n}{s_n} = \frac{6000}{6000} = 1$$

$$h = \frac{6000 \cdot \left( 0.8 + \frac{420}{1400} \right)}{36 + 9 \cdot 1} = 146.77 \text{ mm} > 90 \text{ mm O.K}$$

Use  $h \approx 150$  mm

**c. Slab with beams (5.4 x 4.5) m clear span with  $\alpha_{fm}=0.7$ .**

$$0.2 < \alpha_{fm} = 0.7 < 2$$

$$h = \frac{\ell_n \left( 0.8 + \frac{f_y}{1400} \right)}{36 + 5\beta(\alpha_{fm} - 0.2)}$$

$$\beta = \frac{\ell_n}{s_n} = \frac{5400}{4500} = 1.2$$

$$h = \frac{5400 \cdot \left( 0.8 + \frac{420}{1400} \right)}{36 + 5 \cdot 1.2 \cdot (0.7 - 0.2)} = 152.3 \text{ mm} > 125 \text{ mm O.K}$$

Use  $h \approx 160$  mm

**d. Flat plate (4.6 x 4.6) m clear span.**

From ACI Table 8.3.1.1

$$h = \frac{\ell_n}{33} = \frac{4600}{33} = 139.39 \text{ mm} > 125 \text{ mm O.K}$$

Use  $h \approx 140$  mm ■

**Example 4:** Find the minimum thickness of a slab for an interior panels due to deflection control for the following: Use  $f_y=280$  Mpa (40000 psi)

a. Slab (2.5 x 3.5) m clear span with  $\alpha_{fm}=3.2$

b. Flat plate (6.5 x 7.0) m clear span.

c. Slab without drop panels (6.5 x 5.5 ) m clear span with  $\alpha_{fm}=0.12$

d. Slab with drop panels (6.0 x 7.5) m clear span with  $\alpha_{fm}=0.15$

**Solution:**

**a. Slab (2.5 x 3.5) m clear span with  $\alpha_{fm}=3.2$**

$$\alpha_{fm}=3.2 > 2$$

$$h = \frac{\ell_n(0.8 + \frac{f_y}{1400})}{36 + 9\beta}$$

$$\beta = \frac{\ell_n}{s_n} = \frac{3500}{2500} = 1.4$$

$$h = \frac{3500*(0.8 + \frac{280}{1400})}{36 + 9*1.4} = 72.01 \text{ mm} < 90 \text{ mm Not O.K}$$

Use  $h=90$  mm

**b. Flat plate (6.5 x 7.0) m clear span.**

By using **ACI Table 8.3.1.1**

$$h = \frac{\ell_n}{36} = \frac{7000}{36} = 194.44 \text{ mm} > 125 \text{ O.K}$$

Use  $h \approx 200$  mm

**c. Slab without drop panels (6.5 x 5.5 ) m clear span with  $\alpha_{fm}=0.12$**

$$\alpha_{fm}=0.12 < 0.2$$

Then the slab is considered without beams

By using **ACI Table 8.3.1.1**

$$h = \frac{\ell_n}{36} = \frac{6500}{36} = 180.55 \text{ mm} > 125 \text{ mm O.K}$$

Use  $h \approx 190$  mm

**d. Slab with drop panels (6.0 x 7.5) m clear span with  $\alpha_{fm}=0.15$**

$$\alpha_{fm}=0.15 < 0.2$$

Then the slab is considered without beams

By using **ACI Table 8.3.1.1**

$$h = \frac{\ell_n}{40} = \frac{7500}{40} = 187.5 \text{ mm} > 100 \text{ mm O.K}$$

Use  $h \approx 190$  mm ■

## Home Work

Find the minimum thickness of a slab for an interior panel due to deflection control for the following. Use  $f_y=420$  Mpa (60000 psi)

- a. Flat slab with drop panels (6.4 x 6.0) m clear span
- b. Flat plate (4.4 x 4.0) m clear span.
- c. Slab with beams (5.8 x 5.6) m clear span with with  $\alpha_{fm}=1.7$
- d. Slab with beams (8.0 x 6.5) m clear span with with  $\alpha_{fm}=3.4$
- e. Slab without drop panels (5.5 x 4.8) m clear span with  $\alpha_{fm}=0.19$